

Third Semester B.E. Degree Examination, June/July 2025
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the Laplace transform of :
- i) $\left(\frac{4t+5}{e^{2t}}\right)^2$ ii) $\left(\frac{\sin 2t}{\sqrt{t}}\right)^2$ iii) $t \cos at$. (10 Marks)
- b. The square wave function $f(t)$ with period $2a$ defined by $f(t) = \begin{cases} 1 & 0 \leq t < a \\ -1 & a \leq t < 2a \end{cases}$. Show that $\left(\frac{1}{s}\right) \tanh\left(\frac{as}{2}\right)$. (05 Marks)
- c. Employ Laplace transform to solve $\frac{d^2y}{dt^2} - \frac{dy}{dt} = 0$, $y(0) = y_1(0) = 3$. (05 Marks)

OR

- 2 a. Find (i) $L^{-1}\left\{\frac{s^2-3s+4}{s^3}\right\}$ (ii) $\cot^{-1}\left(\frac{s}{2}\right)$ (iii) $L^{-1}\left\{\frac{s}{(s+2)(s+3)}\right\}$. (10 Marks)
- b. Find the inverse Laplace transform of $\frac{1}{s(s^2+1)}$ using convolution theorem. (05 Marks)
- c. Express $f(t) = \begin{cases} 2 & \text{if } 0 < t < 1 \\ \frac{t^2}{2} & \text{if } 1 < t < \frac{\pi}{2} \\ \cos t & t > \frac{\pi}{2} \end{cases}$ in terms of unit step function and hence find its Laplace transformation. (05 Marks)

Module-2

- 3 a. Obtain fourier series for the function $f(x) = |x|$ in $(-\pi, \pi)$. (06 Marks)
- b. Expand $f(x) = \frac{(\pi-x)^2}{4}$ as a Fourier series in the interval $(0, 2\pi)$ and hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$. (07 Marks)
- c. Express y as a Fourier series upto the second harmonic given :

x:	0	60	120	180	240	300
y:	4	3	2	4	5	6

(07 Marks)

OR

1 of 3

- 4 a. Find the Half-Range sine series of $\pi x - x^2$ in the interval $(0, \pi)$. (06 Marks)
 b. Obtain Fourier expansion of the function $f(x) = 2x - x^2$ in the interval $(0, 3)$. (07 Marks)
 c. Obtain the Fourier expansion of y upto the first harmonic given :

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(07 Marks)

Module-3

- 5 a. If $f(x) = \begin{cases} 1-x^2, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases}$ find the Fourier transform of $f(x)$ and hence find the value of

$$\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} dx.$$
 (07 Marks)
 b. Find the Fourier sine and cosine transform of $f(x) = e^{-\alpha x}$, $\alpha > 0$. (06 Marks)
 c. Solve $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$, given $u_0 = 0$, $u_1 = 1$ by using z-transform. (07 Marks)

OR

- 6 a. Find the Fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx$, $m > 0$. (07 Marks)
 b. Find the Z-transform of $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$. (06 Marks)
 c. Find the inverse Z-transform of

$$\frac{3z^2 + 2z}{(5z-1)(5z+2)}$$
 (07 Marks)

Module-4

- 7 a. Solve $\frac{dy}{dx} = e^x - y$, $y(0) = 2$ using Taylor's Series method upto 4th degree terms and find the value of $y(1.1)$. (07 Marks)
 b. Use Runge-Kutta method of fourth order to solve $\frac{dy}{dx} + y = 2x$ at $x = 1.1$ given $y(1) = 3$ (Take $h = 0.1$). (06 Marks)
 c. Apply Milne's predictor-corrector formulae to compute $y(0.4)$ given $\frac{dy}{dx} = 2e^x y$, with

x	0	0.1	0.2	0.3
y	2.4	2.473	3.129	4.059

(07 Marks)

OR

- 8 a. Given $\frac{dy}{dx} = x + \sin y$; $y(0) = 1$. Compute $y(0.4)$ with $h = 0.2$ using Euler's modified method. (07 Marks)

- b. Apply Runge-Kutta fourth order method, to find $y(0.1)$ with $h = 0.1$ given $\frac{dy}{dx} + y + xy^2 = 0$; $y(0) = 1$. (06 Marks)
- c. Using Adams-Bashforth method, find $y(4.4)$ given $5x\left(\frac{dy}{dx}\right) + y^2 = 2$ with

x	4	4.1	4.2	4.3
y	1	1.0049	1.0097	1.0143

(07 Marks)

Module-5

- 9 a. Using Runge-Kutta method, solve

$$\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2, \text{ for } x = 0.2, \text{ correct to four decimal places, using initial conditions } y(0) = 1, y'(0) = 0. \quad (07 \text{ Marks})$$

- b. Derive Euler's equation in the standard form viz, $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. (07 Marks)

- c. Find the extremal of the functional $\int_{x_1}^{x_2} (y^2 + y'^2 + 2ye^x) dx$. (06 Marks)

OR

- 10 a. Given the differential equation $2\frac{d^2y}{dx^2} = 4x + \frac{dy}{dx}$ and the following table of initial values:

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y'	2	2.3178	2.6725	2.0657

Compute $y(1.4)$ by applying Milne's Predictor-corrector formula. (07 Marks)

- b. Prove that geodesics of a plane surface are straight lines. (07 Marks)

- c. On what curves can the functional $\int_0^1 (y'^2 + 12xy) dx$ with $y(0) = 0$, $y(1) = 1$ can be extremized? (06 Marks)

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