CBCS SCHEME

USN

21MAT21

Second Semester B.E./B.Tech. Degree Examination, June/July 2025
Advanced Calculus and Numerical Methods

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Evaluate :
$$\int_0^2 \int_0^6 \int_0^{4-x^2} dz\ dy\ dx$$
(06 Marks)

b. Change the order of integration and hence evaluate $\int_0^1 \int_{\sqrt{x}}^1 (1+y)\,dy\,dx$ (07 Marks)

c. Define Beta and Gama function. Prove that $\beta(\frac{1}{2}, \frac{1}{2}) = \pi$ (07 Marks)

OR

2 a. Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, using double integration. (06 Marks)

b. Derive the relation between Beta and Gama function. (07 Marks)

c. Find the volume of the tetrahedron in the first octant bounded by $4x + 2y + z = 8$, using double integration. (07 Marks)

Module-2

3 a. A fluid motion is given by $\vec{v} = (y\sin z - \sin x)\hat{i} + (x\sin z + 2yz)\hat{j} + (xy\cos z + y^2)\hat{k}$ show that the motion is irrotational. (06 Marks)

b. Find div($\vec{v}$) and curl($\vec{v}$) of $\vec{v} = (xyz)\hat{i} + (3x^2y)\hat{j} + (xz^2 - y^2z)\hat{k}$ at $(2, -1, 1)$ (07 Marks)

c. Find the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the vector $4\hat{i} - 2\hat{j} + \hat{k}$ (07 Marks)

OR

4 a. If a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ displays a particle in the xy-plane from (0, 0) to (1, 4) along the curve $y = 4x^2$. Find the work done. (06 Marks)

b. Using Green's theorem, evaluate $\int_C (xy - x^2)dx + x^2y\,dy$ where C is bounded by $y = 0$, $x = 1$ and $y = x$. (07 Marks)

c. Evaluate $\int_C \vec{F} \cdot \vec{dr}$ by Stoke's theorem, where $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ and C is the boundary of the rectangle $x = \pm a$, $y = 0$ and $y = b$. (07 Marks)

Module-3

5 a. Form partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2)$. (06 Marks)

b. Solve $\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial y} - 6z = 0$, given that $z = x$ and $\frac{\partial z}{\partial y} = 0$, when $y = 0$. (07 Marks)

c. With usual notations derive a one-dimensional heat equation. (07 Marks)

OR

6 a. Solve : $z = yq - xp$ (06 Marks)

b. Form the partial differential equation from
$Z = f(y + 2x) + g(y - 3x)$ (07 Marks)

c. Solve : $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$ subject to the conditions $z(x, 0) = x^2$ and $z(1, y) = \cos y$. (07 Marks)

Module-4

7 a. Find a real toot of $x \tan x = -1$ in $(2.5, 3)$ by Regula-Falsi method in four iterations. (06 Marks)

b. Apply Newton's general interpolation formula to find u_x. Given that $u_0 = 8$, $u_1 = 11$, $u_4 = 68$, $u_5 = 123$. (07 Marks)

c. Evaluate $\int_0^3 x^4\, dx$ by Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule. (take $n = 6$). (07 Marks)

OR

8 a. Using Newton-Raphson method, find real root of $4x - e^x = 0$ which is near to 2. (06 Marks)

b. Using Simpson's $\left(\frac{3}{8}\right)^{th}$ rule, evaluate $\int_0^{0.3} \sqrt{1 - 8x^3}\, dx$ $(n = 6)$ (07 Marks)

c. In the following table, values of y are consecutive terms of a series of which 23.6 is the 6^{th} term. Find the first and tenth terms of the series:

x	3	4	5	6	7	8	9
y	4.8	8.4	14.5	23.6	36.2	52.8	73.9

(07 Marks)

Module-5

9 a. Employ Taylor's series method, to find $y(0.1)$ from $\frac{dy}{dx} = y + e^{2x}$ with $y(0) = 1$ upto 3^{rd} degree term. (06 Marks)

b. Apply the Runge-Kutta method of 4^{th} order find y at $x = 0.1$. Given that $\frac{dy}{dx} = y + x^3$; $y(0) = 1$, $(h = 0.1)$ (07 Marks)

c. Find y(0.4) using Milne's predictor-corrector method, given $y' = \frac{(1+x^2)y^2}{2}$; y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21 **(07 Marks)**

OR

10 a. Use Tayler's series method to find y(0.1).
Given $\frac{dy}{dx} = y + \sin x$: y(0) = 1 up to the term containing x^3. **(06 Marks)**

b. Using Modified Euler's method find y(0.1), taking h = 0.05, given that
$\frac{dy}{dx} = x^2 + y$: y(0) = 1 **(07 Marks)**

c. Given $\frac{dy}{dx} = \frac{y-x}{y+x}$: y(0) = 1, find y(0.2) taking h = 0.2, using Runge Kutta 4^{th} order method. **(07 Marks)**

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