CBCS SCHEME

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18MAT21

Second Semester B.E. Degree Examination, June/July 2025
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, -1, 2). **(06 Marks)**

b. If $\vec{F} = \nabla(xy^3z^2)$, find div $\vec{F}$ and curl $\vec{F}$ at the point (1, -1, 1). **(07 Marks)**

c. Show that $\vec{F} = (y+z)i + (z+x)j + (x+y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

OR

2 a. If $\vec{F} = xyi + yzj + zxk$, evaluate $\int_c \vec{F}.d\vec{r}$ where c is the curve represented by x = t, y = t^2, z = t^3, $-1 \le t \le 1$. **(06 Marks)**

b. Using the Green's theorem, evaluate $\oint (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region enclosed by $y = \sqrt{x}$ and $y = x^2$. **(07 Marks)**

c. If $\vec{F} = (2x^2 - 32)j - 2xyj - 4xk$ evaluate $\iiint_V \nabla \cdot \vec{F} dv$ where v is the region bounded by the planes x = 0, y = 0, z = 0 and 2x + 2y + z = 4. **(07 Marks)**

Module-2

3 a. Solve $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$ **(06 Marks)**

b. Solve $(D^2 + 1)\, y = \tan x$ by the method of variation of parameter. **(07 Marks)**

c. Solve $x^2y'' - 3xy' + 5y = 3 \sin(\log x)$ **(07 Marks)**

OR

4 a. Solve $(D^2 + 4)y = 2^{-x} + \cos 2x$ **(06 Marks)**

b. Solve $(2x + 1)^2 y'' - 6(2x + 1)y' + 16y = 8(2x + 1)^2$ **(07 Marks)**

c. The differential equation of a simple pendulum $\frac{d^2x}{dt^2} + w^2x = F \sin nt$, where w and F are constants. If at t = 0, x = 0 and $\frac{dx}{dt} = 0$, determine the motion when n = w. **(07 Marks)**

Module-3

5 a. Form the partial differential equation by eliminating the arbitrary function from $lx + my + nz = \phi(x^2 + y^2 + z^2)$ **(06 Marks)**

b. Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x + 3y)$ **(07 Marks)**

c. Derive one-dimensional wave equation in usual notations. **(07 Marks)**

OR

6 a. Solve $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$ (06 Marks)

b. Solve $\frac{\partial^2 z}{\partial y^2} = z$ given that $y = 0$, $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$ (07 Marks)

c. Find the various possible solution of the one dimensional heat equation $u_t = c^2 u_{xx}$ by the method of separation of variable. (07 Marks)

Module-4

7 a. Discuss the nature of the series

$$\sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{n+1}} x^n$$ (06 Marks)

b. With usual notation prove that

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$$ (07 Marks)

c. Express $x^3 - 5x^2 + x + 2$ interms of Legendre's polynomials. (07 Marks)

OR

8 a. Discuss the nature of the series

$$\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \ldots\ldots \quad (x > 0)$$ (06 Marks)

b. Prove the orthogonality property of Bessel's function as $\int_0^1 x\, J_n(\alpha x) J_n(\beta x) dx = 0$, $\alpha \neq \beta$. (07 Marks)

c. If $x^3 + 2x^2 - 4x + 5 = a\,p_3(x) + b\,p_2(x) + c\,p_1(x) + d\,p_0(x)$, find a, b, c and d. (07 Marks)

Module-5

9 a. Using Newton's forward difference formula find f(1.4)

x:	1	2	3	4	5
f(x):	10	26	58	112	194

(06 Marks)

b. Find the real root of $xe^x - \cos x = 0$ correct to three decimal places lying in the interval (·5, ·6) using Regula Falsi-method. (07 Marks)

c. Evaluate $\int_0^1 \frac{x}{1+x^2} dx$ by using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule taking six equal strips. (07 Marks)

OR

10 a. Show that a root of the equation $x^3 + 5x - 11 = 0$ lies between 1 and 2. Find the root by Newton's Raphson method carryout two iterations. (06 Marks)

b. Find f(9) from the data by Newton's divided difference formula.

x:	5	7	11	13	17
y:	150	392	1452	2366	5202

(07 Marks)

c. Evaluate $\int_4^{5.2} \log_e x\, dx$ taking six equal strips by applying Weddle's rule. (07 Marks)

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