

Second Semester B.E./B.Tech. Degree Examination, June/July 2025 Mathematics-II for ME Stream

Time: 3 hrs.

Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.

2. M : Marks, L: Bloom's level, C: Course outcomes.

3. VTU Formula Hand Book is permitted

Module - 1				M	L	C
Q.1	a.	Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dy dx$.		7	L3	CO1
	b.	Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) dy dx dz$.		7	L3	CO1
	c.	Show that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.		6	L2	CO1
OR						
Q.2	a.	Evaluate by changing into polar coordinates. $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$.		7	L3	CO1
	b.	Find the area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, by double integration.		7	L2	CO1
	c.	Write a modern mathematical tool program to find the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.		6	L3	CO5
Module - 2						
Q.3	a.	Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$.		7	L2	CO2
	b.	Define a irrotational vector. Find the constants a, b and c such that the vector $\vec{F} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$ is irrotational.		7	L2	CO2
	c.	If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$.		6	L2	CO2
OR						
Q.4	a.	Find the workdone by a force $\vec{F} = (2y - x^2)\hat{i} + 6yz\hat{j} - 8xz^2\hat{k}$ from the point $(0, 0, 0)$ to the point $(1, 1, 1)$ along the straight line joining these points.		7	L2	CO2
	b.	Using Green's theorem, evaluate $\int_C (xy + y^2) dx + x^2 dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.		7	L3	CO2

	c.	Write the modern mathematical tool program to find the divergence of the vector field $\vec{F} = x^2yz \hat{i} + y^2zx \hat{j} + z^2xy \hat{k}$.	6	L3	CO5										
Module – 3															
Q.5	a.	From the PDE by eliminating the arbitrary function from $f(x + y + z, x^2 + y^2 + z^2) = 0$.	7	L2	CO3										
	b.	Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that when $x = 0$, $z = 0$ and $\frac{\partial z}{\partial x} = a \sin y$.	7	L3	CO3										
	c.	Derive one dimensional heat equation in the standard form as $\frac{\partial u}{\partial t} = C^2 \cdot \frac{\partial^2 u}{\partial x^2}$.	6	L2	CO3										
OR															
Q.6	a.	Form the PDE by eliminating the arbitrary constants 'a' and 'b' from $(x - a)^2 + (y - b)^2 + z^2 = c^2$.	7	L2	CO3										
	b.	Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2 \sin y$ when $x = 0$ and $z = 0$, if 'y' is an odd multiple of $\pi/2$.	7	L3	CO3										
	c.	Solve : $x \cdot \frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y} = z$, using Lagrange's multipliers.	6	L3	CO3										
Module – 4															
Q.7	a.	Find the real root of the equation $xe^x - 2 = 0$ correct to three decimal places using the Newton – Raphson method.	7	L3	CO4										
	b.	Using Newton's forward interpolation formula, find y at $x = 5$. <table><tr><td>x</td><td>4</td><td>6</td><td>8</td><td>10</td></tr><tr><td>y</td><td>1</td><td>3</td><td>8</td><td>16</td></tr></table>	x	4	6	8	10	y	1	3	8	16	7	L3	CO4
x	4	6	8	10											
y	1	3	8	16											
	c.	Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ by taking 7 ordinates using the Trapezoidal rule.	6	L3	CO4										
OR															
Q.8	a.	Compute the real root of the equation $x \log_{10} x - 1.2 = 0$ lies between 2 and 3 by the Regula Falsi method. Carry out four approximations.	7	L2	CO4										
	b.	Using Newton's divided difference formula, evaluate $f(4)$ from the following table : <table><tr><td>x</td><td>0</td><td>2</td><td>3</td><td>6</td></tr><tr><td>f(x)</td><td>-4</td><td>2</td><td>14</td><td>158</td></tr></table>	x	0	2	3	6	f(x)	-4	2	14	158	7	L2	CO4
x	0	2	3	6											
f(x)	-4	2	14	158											
	c.	Compute the value of y when $x = 3$. Using Lagrange's interpolation formula given <table><tr><td>x</td><td>-2</td><td>-1</td><td>1</td><td>2</td></tr><tr><td>y</td><td>-7</td><td>2</td><td>0</td><td>11</td></tr></table>	x	-2	-1	1	2	y	-7	2	0	11	6	L3	CO4
x	-2	-1	1	2											
y	-7	2	0	11											

Module – 5					
Q.9	a.	Use Taylor's series method to find y at x = 0.1 considering the terms upto the third degree given that $\frac{dy}{dx} = x^2 + y^2$ and y(0) = 1.	7	L3	CO4
	b.	Apply the Runge – Kutta method of fourth order to find an approximate value of y at x = 0.2, given that $\frac{dy}{dx} = 3x + \frac{y}{2}$ with y(0) = 1, and h = 0.2.	7	L3	CO4
	c.	Given that $\frac{dy}{dx} = x - y^2$ and the data y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762. Compute y at x = 0.8 by applying Milne's method.	6	L3	CO4
OR					
Q.10	a.	Using the modified Euler's method, find y(0.1) given that $\frac{dy}{dx} = x^2 + y$ and y(0) = 1 take step size h = 0.05 and perform two approximations in each stage.	7	L3	CO4
	b.	Using the Runge-Kutta method of fourth order, find y(0.2) given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0) = 1, taking h = 0.2.	7	L3	CO4
	c.	Using modern mathematical tools write a program to find y when x = 2 given that $\frac{dy}{dx} = 1 + \frac{y}{x}$, y(1) = 2, taking h = 0.2 by Runge – Kutta method of 4 th order.	6	L3	CO5
