

## Second Semester B.E./B.Tech. Degree Examination, June/July 2025 Mathematics – II for EEE stream

Time: 3 hrs.

Max. Marks: 100

- Note:** 1. Answer any FIVE full questions, choosing ONE full question from each module.  
2. VTU Formula Hand Book is permitted.  
3. M : Marks, L: Bloom's level, C: Course outcomes.

| Module – 1 |    |                                                                                                                                                                                                         | M | L  | C   |
|------------|----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|----|-----|
| Q.1        | a. | Find the angle between the surface $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point $(1, -2, 1)$                                                                                               | 7 | L3 | CO1 |
|            | b. | Evaluate $\text{curl}(\text{curl } \vec{F})$ and $\text{div}(\text{curl } \vec{F})$ if $\vec{F} = x^2yi + y^2zj + z^2xk$ .                                                                              | 7 | L3 | CO1 |
|            | c. | Show that the vector $\vec{F} = \frac{xi + yj}{x^2 + y^2}$ is both solenoidal and irrotational.                                                                                                         | 6 | L2 | CO2 |
| OR         |    |                                                                                                                                                                                                         |   |    |     |
| Q.2        | a. | Find the total work done by the force $\vec{F} = 3xyi - yj + 2zxk$ in moving a particle around the circle $x^2 + y^2 = 4$ .                                                                             | 7 | L3 | CO1 |
|            | b. | Using Green's theorem evaluate $\oint_C (xy + y^2)dx + x^2dy$ over the region bounded by the curves $y = x$ and $y = x^2$                                                                               | 7 | L2 | CO3 |
|            | c. | Using modern mathematical tools, write the code to find gradient of $\phi = x^2y + 2xz - 4$ .                                                                                                           | 6 | L2 | CO5 |
| Module – 2 |    |                                                                                                                                                                                                         |   |    |     |
| Q.3        | a. | Define a Subspace. Show that any plane passing through the origin is a subspace of $R^3$ .                                                                                                              | 7 | L1 | CO1 |
|            | b. | Let V be the vector space of all real valued continuous functions over R. Show that the set W of solutions of differential equations $5\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 2y = 0$ is a subspace of V. | 7 | L2 | CO2 |
|            | c. | Define a Inner Product space. Consider $f(t) = 3t - 5$ and $g(t) = t^2$ , the inner product $\langle f, g \rangle = \int_0^t f(t)g(t)dt$ . Find $\langle f, g \rangle$                                  | 6 | L2 | CO2 |
| OR         |    |                                                                                                                                                                                                         |   |    |     |
| Q.4        | a. | Express the vector $(3, 5, 2)$ as a linear combination of the vectors $(1, 1, 0)$ , $(2, 3, 0)$ , $(0, 0, 1)$ of $V_3(R)$ .                                                                             | 7 | L2 | CO3 |

|            |     |                                                                                                                                                                                                                                                             |     |     |     |     |    |    |    |    |     |     |     |     |     |     |   |    |     |
|------------|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|-----|-----|-----|----|----|----|----|-----|-----|-----|-----|-----|-----|---|----|-----|
|            | b.  | State Rank-Nullity theorem. Verify the Rank-Nullity theorem for the $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (x+2y-z, y+z, x+y-2z)$                                                                                              | 7   | L2  | CO2 |     |    |    |    |    |     |     |     |     |     |     |   |    |     |
|            | c.  | Using the modern mathematical tool, write the code to represent the reflection transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ and to find the image of vector $(10, 0)$ when it is reflected about the y-axis.                                   | 6   | L1  | CO4 |     |    |    |    |    |     |     |     |     |     |     |   |    |     |
| Module – 3 |     |                                                                                                                                                                                                                                                             |     |     |     |     |    |    |    |    |     |     |     |     |     |     |   |    |     |
| Q.5        | a.  | Find the Laplace transform of, (i) $e^{-3t}(2\cos 5t - 3\sin 5t)$<br>(ii) $\frac{\cos at - \cos bt}{t}$                                                                                                                                                     | 7   | L3  | CO3 |     |    |    |    |    |     |     |     |     |     |     |   |    |     |
|            | b.  | Given $f(t) = \begin{cases} E, & 0 < t < \frac{a}{2} \\ -E, & \frac{a}{2} < t < a \end{cases}$ where $f(t+a) = f(t)$ , show that $L[f(t)] = \frac{E}{s} \tanh\left(\frac{as}{4}\right)$                                                                     | 7   | L2  | CO2 |     |    |    |    |    |     |     |     |     |     |     |   |    |     |
|            | c.  | Express $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$ in terms of the Heaviside unit step function and hence find $L[f(t)]$ .                                                                | 6   | L3  | CO4 |     |    |    |    |    |     |     |     |     |     |     |   |    |     |
| OR         |     |                                                                                                                                                                                                                                                             |     |     |     |     |    |    |    |    |     |     |     |     |     |     |   |    |     |
| Q.6        | a.  | Find $L^{-1}\left[\frac{2s^2 + 5s - 4}{s^3 + s^2 - 2s}\right]$                                                                                                                                                                                              | 7   | L3  | CO3 |     |    |    |    |    |     |     |     |     |     |     |   |    |     |
|            | b.  | Using the convolution theorem, find the inverse Laplace Transform of $\frac{s}{(s^2 + a^2)^2}$ .                                                                                                                                                            | 7   | L3  | CO4 |     |    |    |    |    |     |     |     |     |     |     |   |    |     |
|            | c.  | Solve $y'' + 2y' - y' - 2y = 0$ , given $y(0) = y'(0) = 0$ and $y''(0) = 6$ by using Laplace transform method.                                                                                                                                              | 6   | L2  | CO3 |     |    |    |    |    |     |     |     |     |     |     |   |    |     |
| Module – 4 |     |                                                                                                                                                                                                                                                             |     |     |     |     |    |    |    |    |     |     |     |     |     |     |   |    |     |
| Q.7        | a.  | By Newton-Raphson method, find the root of $x \tan x + 1 = 0$ which is near to $x = \pi$ .                                                                                                                                                                  | 7   | L3  | CO2 |     |    |    |    |    |     |     |     |     |     |     |   |    |     |
|            | b.  | Using Newton's forward difference formula find $f(38)$ ,<br><table><tr><td>x</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td></tr><tr><td>y</td><td>184</td><td>204</td><td>226</td><td>250</td><td>276</td><td>304</td></tr></table> | x   | 40  | 50  | 60  | 70 | 80 | 90 | y  | 184 | 204 | 226 | 250 | 276 | 304 | 7 | L3 | CO3 |
| x          | 40  | 50                                                                                                                                                                                                                                                          | 60  | 70  | 80  | 90  |    |    |    |    |     |     |     |     |     |     |   |    |     |
| y          | 184 | 204                                                                                                                                                                                                                                                         | 226 | 250 | 276 | 304 |    |    |    |    |     |     |     |     |     |     |   |    |     |
|            | c.  | Use Lagrange's interpolation formula to find $y$ at $x = 10$ , given<br><table><tr><td>x</td><td>5</td><td>6</td><td>9</td><td>11</td></tr><tr><td>y</td><td>12</td><td>13</td><td>14</td><td>16</td></tr></table>                                          | x   | 5   | 6   | 9   | 11 | y  | 12 | 13 | 14  | 16  | 6   | L3  | CO3 |     |   |    |     |
| x          | 5   | 6                                                                                                                                                                                                                                                           | 9   | 11  |     |     |    |    |    |    |     |     |     |     |     |     |   |    |     |
| y          | 12  | 13                                                                                                                                                                                                                                                          | 14  | 16  |     |     |    |    |    |    |     |     |     |     |     |     |   |    |     |

OR

OR

|     |                                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                     |     |     |     |      |   |   |    |   |    |    |     |     |     |      |   |    |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|-----|-----|------|---|---|----|---|----|----|-----|-----|-----|------|---|----|
| Q.8 | a.                                                                                                                                               | Find a real root of the equation $x^3 - 2x - 5 = 0$ correct to three decimal places by the method of false position in (2, 3).                                                                                                                                                                                                                      | 7   | L4  | CO4 |      |   |   |    |   |    |    |     |     |     |      |   |    |
|     | b.                                                                                                                                               | Find the interpolating polynomial using Newton's dividend difference formula for the following data,<br><table border="1" style="margin: 10px auto; width: 60%;"><tr><td>x</td><td>2</td><td>4</td><td>5</td><td>6</td><td>8</td><td>10</td></tr><tr><td>y</td><td>10</td><td>96</td><td>196</td><td>350</td><td>868</td><td>1746</td></tr></table> | x   | 2   | 4   | 5    | 6 | 8 | 10 | y | 10 | 96 | 196 | 350 | 868 | 1746 | 7 | L3 |
| x   | 2                                                                                                                                                | 4                                                                                                                                                                                                                                                                                                                                                   | 5   | 6   | 8   | 10   |   |   |    |   |    |    |     |     |     |      |   |    |
| y   | 10                                                                                                                                               | 96                                                                                                                                                                                                                                                                                                                                                  | 196 | 350 | 868 | 1746 |   |   |    |   |    |    |     |     |     |      |   |    |
| c.  | Use Simpson's $\frac{3}{8}$ rule to obtain the approximate value of $\int_0^{0.3} (1 - 8x^3)^{\frac{1}{2}} dx$ by considering 3 equal intervals. | 6                                                                                                                                                                                                                                                                                                                                                   | L2  | CO2 |     |      |   |   |    |   |    |    |     |     |     |      |   |    |

Module - 5

|     |    |                                                                                                                                                                                                                   |   |    |     |
|-----|----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|----|-----|
| Q.9 | a. | Use Taylor's series method, find $y(0.1)$ considering upto fourth degree term if $y(x)$ satisfies the equation, $\frac{dy}{dx} = x - y^2$ , $y(0) = 1$ .                                                          | 7 | L3 | CO3 |
|     | b. | Given $\frac{dy}{dx} = 3x + \frac{y}{2}$ , $y(0) = 1$ . Compute $y(0.2)$ by taking $h = 0.2$ , using Runge-Kutta method of fourth order.                                                                          | 7 | L2 | CO3 |
|     | c. | Apply Milne's method to compute $y(1.4)$ , correct to four decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and following the data $y(1) = 2$ , $y(1.1) = 2.2156$ , $y(1.2) = 2.4649$ , $y(1.3) = 2.7514$ | 6 | L2 | CO2 |

OR

|      |    |                                                                                                                                                                                    |   |    |     |
|------|----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|----|-----|
| Q.10 | a. | Using modified Euler's method to find $y$ at $x = 0.2$ given $\frac{dy}{dx} = x - y^2$ and $y(0) = 1$ by taking step size $h = 0.1$ .                                              | 7 | L4 | CO4 |
|      | b. | Using the Runge-Kutta method of fourth order find $y(0.1)$ given that $\frac{dy}{dx} = 3e^x + 2y$ , $y(0) = 0$ taking $h = 0.1$ .                                                  | 7 | L3 | CO2 |
|      | c. | Using modern mathematical tools, write the code to find the solution of $\frac{dy}{dx} = x - y^2$ at $y(0.1)$ , given that $y(0) = 1$ by Runge-Kutta 4 <sup>th</sup> order method. | 6 | L2 | CO3 |

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