

Second Semester B.E./B.Tech. Degree Examination, June/July 2025
Advanced Calculus and Numerical Methods

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Evaluate :

$$\int_0^2 \int_0^6 \int_0^{4-x^2} dz \, dy \, dx$$

(06 Marks)

- b. Change the order of integration and hence evaluate
- $\int_0^1 \int_{\sqrt{x}}^1 (1+y) dy dx$

(07 Marks)

- c. Define Beta and Gamma function. Prove that
- $\beta(\frac{1}{2}, \frac{1}{2}) = \pi$

(07 Marks)

OR

- 2 a. Find the area between the parabolas
- $y^2 = 4ax$
- and
- $x^2 = 4ay$
- , using double integration.

(06 Marks)

- b. Derive the relation between Beta and Gamma function.

(07 Marks)

- c. Find the volume of the tetrahedron in the first octant bounded by
- $4x + 2y + z = 8$
- , using double integration.

(07 Marks)

Module-2

- 3 a. A fluid motion is given by
- $\vec{v} = (y \sin z - \sin x)\hat{i} + (x \sin z + 2yz)\hat{j} + (xy \cos z + y^2)\hat{k}$
- show that the motion is irrotational.

(06 Marks)

- b. Find
- $\text{div}(\vec{v})$
- and
- $\text{curl}(\vec{v})$
- of
- $\vec{v} = (xyz)\hat{i} + (3x^2y)\hat{j} + (xz^2 - y^2z)\hat{k}$
- at
- $(2, -1, 1)$

(07 Marks)

- c. Find the directional derivative of the function
- $\phi = x^2 - y^2 + 2z^2$
- at the point
- $P(1, 2, 3)$
- in the direction of the vector
- $4\hat{i} - 2\hat{j} + \hat{k}$

(07 Marks)

OR

- 4 a. If a force
- $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$
- displays a particle in the xy-plane from
- $(0, 0)$
- to
- $(1, 4)$
- along the curve
- $y = 4x^2$
- . Find the work done.

(06 Marks)

- b. Using Green's theorem, evaluate
- $\int_C (xy - x^2)dx + x^2y dy$
- where
- C
- is bounded by
- $y = 0$
- ,
- $x = 1$
- and
- $y = x$
- .

(07 Marks)

- c. Evaluate
- $\int_C \vec{F} \cdot d\vec{r}$
- by Stoke's theorem, where
- $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$
- and
- C
- is the boundary of the rectangle
- $x = \pm a$
- ,
- $y = 0$
- and
- $y = b$
- .

(07 Marks)



Module-3

- 5 a. Form partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2)$. (06 Marks)
- b. Solve $\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial y} - 6z = 0$, given that $z = x$ and $\frac{\partial z}{\partial y} = 0$, when $y = 0$. (07 Marks)
- c. With usual notations derive a one-dimensional heat equation. (07 Marks)

OR

- 6 a. Solve : $z = yq - xp$ (06 Marks)
- b. Form the partial differential equation from $Z = f(y + 2x) + g(y - 3x)$ (07 Marks)
- c. Solve : $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$ subject to the conditions $z(x, 0) = x^2$ and $z(1, y) = \cos y$. (07 Marks)

Module-4

- 7 a. Find a real root of $x \tan x = -1$ in $(2.5, 3)$ by Regula-Falsi method in four iterations. (06 Marks)
- b. Apply Newton's general interpolation formula to find u_x . Given that $u_0 = 8, u_1 = 11, u_4 = 68, u_5 = 123$. (07 Marks)
- c. Evaluate $\int_0^3 x^4 dx$ by Simpson's $\left(\frac{1}{3}\right)^{th}$ rule. (take $n = 6$). (07 Marks)

OR

- 8 a. Using Newton-Raphson method, find real root of $4x - e^x = 0$ which is near to 2. (06 Marks)
- b. Using Simpson's $\left(\frac{3}{8}\right)^{th}$ rule, evaluate $\int_0^{0.3} \sqrt{1-8x^3} dx$ ($n = 6$) (07 Marks)
- c. In the following table, values of y are consecutive terms of a series of which 23.6 is the 6th term. Find the first and tenth terms of the series:

x	3	4	5	6	7	8	9
y	4.8	8.4	14.5	23.6	36.2	52.8	73.9

(07 Marks)

Module-5

- 9 a. Employ Taylor's series method, to find $y(0.1)$ from $\frac{dy}{dx} = y + e^{2x}$ with $y(0) = 1$ upto 3rd degree term. (06 Marks)
- b. Apply the Runge-Kutta method of 4th order find y at $x = 0.1$. Given that $\frac{dy}{dx} = y + x^3$; $y(0) = 1, (h = 0.1)$ (07 Marks)

- c. Find $y(0.4)$ using Milne's predictor-corrector method, given $y' = \frac{(1+x^2)y^2}{2}$; $y(0) = 1$, $y(0.1) = 1.06$, $y(0.2) = 1.12$, $y(0.3) = 1.21$ (07 Marks)

OR

- 10 a. Use Taylor's series method to find $y(0.1)$.

Given $\frac{dy}{dx} = y + \sin x$; $y(0) = 1$ up to the term containing x^3 . (06 Marks)

- b. Using Modified Euler's method find $y(0.1)$, taking $h = 0.05$, given that

$\frac{dy}{dx} = x^2 + y$; $y(0) = 1$ (07 Marks)

- c. Given $\frac{dy}{dx} = \frac{y-x}{y+x}$; $y(0) = 1$, find $y(0.2)$ taking $h = 0.2$, using Runge Kutta 4th order method. (07 Marks)
