



CBCS SCHEME

18MAT11

First Semester B.E./B.Tech. Degree Examination, June/July 2025 Calculus and Linear Algebra

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Show that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersect orthogonally. (06 Marks)
- b. Find the radius of curvature of the curve $y = a \log \sec \left(\frac{x}{a} \right)$ at any point (x, y) . (06 Marks)
- c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$. (08 Marks)

OR

- 2 a. With usual notation, prove that $\tan \phi = r \frac{d\theta}{dr}$. (06 Marks)
- b. Find the pedal equation of the curve $r = ac^{\theta \cot \alpha}$. (06 Marks)
- c. Find the radius of curvature for the curve $r = a(1 + \cos \theta)$. (08 Marks)

Module-2

- 3 a. Using Maclaurin's series, prove that $\sqrt{1 + \sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \dots$ (08 Marks)
- b. Evaluate $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{\frac{1}{x}}$. (06 Marks)
- c. Examine the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ for its extreme values. (06 Marks)

OR

- 4 a. If $U = f(x - y, y - z, z - x)$, prove that $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} = 0$. (06 Marks)
- b. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (07 Marks)
- c. A rectangular box, open at the top, is to have a volume of 32 cubic ft. Find the dimension of the box requiring least material for its construction. (07 Marks)

Module-3

- 5 a. Evaluate $\int_0^1 \int_{y^2}^{1-y} \int_0^{1-x} x \, dz \, dx \, dy$. (07 Marks)
- b. Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, above x-axis. (07 Marks)
- c. With usual notations, prove that $\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$. (06 Marks)

OR

- 6 a. Evaluate $\int_0^a \int_y^a \frac{x}{x^2 + y^2} dx dy$ by changing the order of integration. (07 Marks)
- b. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing into polar coordinates. (07 Marks)
- c. Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin \theta} d\theta \times \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin \theta}} = \pi$. (06 Marks)

Module-4

- 7 a. Solve $\frac{dy}{dx} + y \tan x = y^2 \sec x$. (06 Marks)
- b. Find the orthogonal trajectories of the family $r^n \cos n\theta = a^n$. (07 Marks)
- c. Solve the equation $(px - y)(py + x) = 2p$ by reducing into Clairaut's form, taking the substitution $X = x^2, Y = y^2$. (07 Marks)

OR

- 8 a. If the temperature of the air is 30°C and a metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach a temperature of 40°C . (06 Marks)
- b. Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$, where λ is the parameter. (07 Marks)
- c. Solve $xy \left(\frac{dy}{dx} \right)^2 - (x^2 + y^2) \frac{dy}{dx} + xy = 0$. (07 Marks)

Module-5

- 9 a. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ by reducing to row-reduced echelon form. (06 Marks)
- b. Apply Gauss-elimination method to solve the $x + 4y - z = -5$, $x + y - 6z = -12$, $3x - y - z = 4$. (07 Marks)
- c. Find numerically largest eigen value and corresponding eigen vector of $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by

Rayleigh's power method. Take initial eigen vector $[1, 0, 0]^T$. Carry out five iterations.

(07 Marks)

OR

- 10 a. Test for consistency and solve the system of equations, $x + y + z = 6$, $x - y + 2z = 5$, $3x + y + z = 8$. (06 Marks)
- b. Solve the system of equations by Gauss-Seidel method $x + y + 54z = 110$, $27x + 6y - z = 85$, $6x + 15y + 2z = 72$. Carryout three iterations. (07 Marks)
- c. Diagonalize the matrix $\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. (07 Marks)
