CBCS SCHEME - Summer Semester

USN | | | | | | | | | | |

21MAT11

First Semester B.E/B.Tech. Degree Examination, June/July 2025
Calculus and Differential Equations

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Prove with usual notations $\tan\phi = \frac{d\theta}{dr}$. **(06 Marks)**

b. Show that curves intersect orthogonally $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ **(07 Marks)**

c. Find the radius of curvature for the Folium of De-cartes $x^3+y^3 = 3axy$ at the point $(3a/2, 3a/2)$ on it. **(07 Marks)**

OR

2 a. If p be the perpendicular from the pole on the tangent, then show that
$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left[\frac{dr}{d\theta}\right]^2$$ **(06 Marks)**

b. Find the pedal equation of the curve $\frac{2a}{r} = (1+\cos\theta)$. **(07 Marks)**

c. Find the radius of curvature for the curve $x^2y = a(x^2+y^2)$ **(07 Marks)**

Module-2

3 a. Using Maclaurin's Series, prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24}$........ **(06 Marks)**

b. If $\mu = f\left[\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right]$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ **(07 Marks)**

c. Find the extreme values of the function $f(x,y) = x^3+y^3-3x-12y+20$. **(07 Marks)**

OR

4 a. Evaluate $\lim_{x \to 0}\left[\frac{\tan x}{x}\right]^{1/x}$ **(06 Marks)**

b. If $u = e^{ax+by} f(ax-by)$ prove that $b\frac{\partial u}{\partial x} + a\frac{\partial u}{\partial y} = 2abu$ **(07 Marks)**

c. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ show that $\frac{\partial(u,v,\omega)}{\partial(x,y,z)} = 4$. **(07 Marks)**

Module-3

5 a. Solve $\frac{dy}{dx}+\frac{y}{x}=xy^2$ (06 Marks)

b. Water at temperature 10°C takes 5 minutes to warm upto 20°C at a room temperature of 40°C. Find the temperature of water after 20 minutes. (07 Marks)

c. Solve $xyp^2-(x^2+y^2)p+xy=0$. (07 Marks)

OR

6 a. Solve : $y(2xy+1)dx - xdy=0$. (06 Marks)

b. Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$, Where λ is the parameter. (07 Marks)

c. Find the general solution of the equation $(px-y)(py+x)=a^2p$ by reducing into Clairaut's form, taking the substitution $X=x^2$, $Y=y^2$. (07 Marks)

Module-4

7 a. Solve : $\frac{d^2y}{dx^2}-4y=\cosh(2x-1)+3^x$ (06 Marks)

b. Solve: $y''+2y'+y=2x+x^2$. (07Marks)

c. Solve $\frac{d^2y}{dx^2}+y=\sec x\tan x$ by the method of variation of parameters. (07 Marks)

OR

8 a. Solve : $\frac{d^3y}{dx^3}+y=65\cos(2x+1)$ (06 Marks)

b. Solve the differential equation $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+25y=e^{2x}+\sin x$ (07 Marks)

c. Solve: $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\cos(\log x)$ (07 Marks)

Module-5

9 a. Find the rank of matrix

$$A=\begin{bmatrix}2&-1&-3&-1\\1&2&3&-1\\1&0&1&1\\0&1&1&-1\end{bmatrix}$$ (06 Marks)

b. Solve the system of equations by using the Gauss Jordan method.

$x+y+z=9$

$x+2y+3z=8$

$2x+y-z=3$ (07 Marks)

c. Using Rayleigh's power method find the dominant eigenvalue and the corresponding eigen vector of $\begin{bmatrix}4&1&-1\\2&3&-1\\-2&1&5\end{bmatrix}$ by taking $[1\quad 0\quad 0]^T$ as initial eigen vector (carry out 6 iterations). (07 Marks)

OR

10 a. Investigate the values of λ and μ such that the system of equations.

$x+y+z=6$

$x+2y+3z=10$

$x+2y+\lambda z=\mu$ may have

i) Unique solution

ii) Infinite solution

iii) No solution

(06 Marks)

b. Apply Gauss elimination method to solve the system of equations.

$2x+5y+7z=52$

$2x+y-z=0$

$x+y+z=9$

(07 Marks)

c. Solve the following system of equations by Gauss- Seidel method.

$20x+y-2z=17$

$3x+20y-z=-18$

$2x-3y+20z=25$

(07 Marks)

* * * * *