15MAT21

Second Semester B.E. Degree Examination, June/July 2025 Engineering Mathematics – II

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Solve
$$(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$$

(05 Marks)

b. Solve
$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x} \sin 2x$$

(05 Marks)

c. Solve
$$\frac{d^2y}{dx^2} + y = \sec x \tan x$$
 by the method of variation of parameters.

(06 Marks)

OR

2 a. Solve
$$y'' - 2y' + y = x \cos x$$

(05 Marks)

b. Solve
$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = e^x + 1$$

(05 Marks)

c. Solve
$$y'' - 5y' + 6y = e^{3x} + x$$
 by the method of undetermined coefficients.

(06 Marks)

Module-2

3 a. Solve
$$x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = 1 + x^2$$

(05 Marks)

b. Solve
$$\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$$

(05 Marks)

c. Solve the equation (px-y) (x-py) = 2p by using the substitution
$$x^2 = X$$
 and $y^2 = Y$. (06 Marks)

OR

4 a. Solve
$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin 2[\log(1+x)]$$

(05 Marks)

b. Solve
$$y = 3px + 6p^2y^2$$
 by solving for x.

(05 Marks)

c. Find the general and singular solution of
$$(y - px) (p - 1) = p$$
.

(06 Marks)

Module-3

5 a. Obtain the partial differential equation by eliminating the arbitrary function given $Z = y f(x) + x \phi(y)$. (05 Marks)

b. Solve $\frac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x - y) = 0$.

(05 Marks)

c. Derive one dimensional heat equation
$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$
.

(06 Marks)

2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice. Important Note: 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.

- 6 a. Obtain the partial differential equation by eliminating the arbitrary function from $\phi(x^2+y^2,\,z-xy)=0\,. \tag{05 Marks}$
 - b. Solve $\frac{\partial^2 z}{\partial y^2} + z = 0$ given that $z = \cos x$ and $\frac{\partial z}{\partial y} = \sin x$ when y = 0. (05 Marks)
 - c. Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables for constant K = 0.

Module-4

7 a. Evaluate
$$\int_{-1}^{1} \int_{x-z}^{z} \int_{x-z}^{x+z} (x+y+z) \, dy \, dx \, dz$$
 (05 Marks)

- b. Evaluate $\int_{0}^{1} \int_{x}^{\sqrt{x}} xy \, dy \, dx$ by changing the order of integration. (05 Marks)
- c. Evaluate $\int_{0}^{2} \frac{x^2}{\sqrt{2-x}} dx$ by using Gamma and Beta functions. (06 Marks)

OR

- 8 a. Evaluate $\int_{0}^{a} \int_{0}^{\sqrt{a^2-y^2}} y\sqrt{x^2+y^2} dx dy by changing into polar coordinates.$ (05 Marks)
 - b. Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integrals in first octant. (05 Marks)
 - c. Obtain the relation between beta and gamma functions in the form $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

 (06 Marks)

Module-5

- 9 a. Find: i) $L(t^5e^{4t}\cosh 3t)$ and ii) $L(\frac{e^{-at}-e^{-bt}}{t})$. (05 Marks)
 - b. If f(t) is a periodic function of period T, then show that $L\{f(t)\} = \frac{1}{1 e^{-st}} \int_{0}^{T} e^{-st} f(t) dt$ (05 Marks)
 - c. Express $f(t) = \begin{cases} t^2 & 0 < t \le 2 \\ 4t & t \ge 2 \end{cases}$ in terms of unit step function hence find its Laplace transformation. (06 Marks)

OR

- 10 a. Find $L^{-1}\left\{\frac{2s-1}{s^2+4s+29}\right\}$ (05 Marks)
 - b. Find $L^{-1} \left[\frac{s}{(s^2 + a^2)^2} \right]$ by using convolution theorem. (05 Marks)
 - c. Solve using Laplace transformation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + y = 0$ under the given conditions y(0) = 1 y'(0) = 0. (06 Marks)