CBCS SCHEME

USN | | | | | | | | | | 21MAT11

First Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025

Calculus and Differential Equations

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. With usual notations prove that
$$\tan\phi = r\frac{d\theta}{dr}$$ **(06 Marks)**

b. Show that curves $\frac{2a}{r} = 1+\cos\theta$ and $\frac{2a}{r} = 1-\cos\theta$ cuts orthogonally. **(07 Marks)**

c. Prove that for the Cardioids $r = a(1+\cos\theta)$, $\frac{\rho^2}{r}$ is constant. **(07 Marks)**

OR

2 a. If p be the perpendicular from the pole on the tangent, then show that
$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$$ **(06 Marks)**

b. Find the pedal equation of the curve $r^n = a^n\cos n\theta$ **(07 Marks)**

c. Find the radius of curvature of the curve
$x^3 + y^3 = 3xy$ at (3/2, 3/2) **(07 Marks)**

Module-2

3 a. Expand $\log(1+\sin x)$ by Maclaurin's series upto the term containing x^4. **(06 Marks)**

b. If $u = \tan^{-1}\left(\frac{x}{y}\right)$, where $x = e^t - e^{-t}$ and $y = e^t + e^{-t}$, find the total derivative $\frac{du}{dt}$. **(07 Marks)**

c. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, –1, 0). **(07 Marks)**

OR

4 a. Evaluate $\lim\limits_{x\to 0}\left(\frac{a^x + b^x + c^x + d^x}{4}\right)^{1/x}$ **(06 Marks)**

b. If $u = f(y-z, z-x, x-y)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(07 Marks)**

c. Examine the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ for extreme values. **(07 Marks)**

Module-3

5 a. Solve $\left[1+e^{x/y}\right]dx + e^{x/y}\left[1-\frac{x}{y}\right]dy = 0$ **(06 Marks)**

b. Show that the family of curves $y^2 = 4a(x+a)$ is self orthogonal. **(07 Marks)**

c. Solve $x^2\left(\frac{dy}{dx}\right)^2 + xy\frac{dy}{dx} - 6y^2 = 0$ **(07 Marks)**

OR

6 a. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2x$. **(06 Marks)**

b. A body originally at 80°C cools down to 60°C in 20 minutes in the surroundings of temperature 40°C. Find the temperature of the body after 40 minutes from the original instant. **(07 Marks)**

c. Find the general and singular solution of $y = xp + \sqrt{4+p^2}$ **(07 Marks)**

Module-4

7 a. Solve $(D^4 + 4D^3 - 5D^2 - 36D - 36)\,y = 0$. **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = e^{2x} + \sin x$ **(07 Marks)**

c. Solve $\frac{d^2y}{dx^2} + y = \sec x$ by using method of variation of parameters. **(07 Marks)**

OR

8 a. Solve $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = e^{2x} + e^x$ **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x^2 + 3x + 1$ **(07 Marks)**

c. Solve $x^2y'' - 4xy' - 6y = \cos(2\log x)$ **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

by using elementary row transformation. **(06 Marks)**

b. Solve the system of equations by using Gauss - Jordan method.
$x + 2y + z = 8$, $2x + 3y + 4z = 20$, $4x + 3y + 2z = 16$ **(07 Marks)**

c. Find the largest eigen values and the corresponding eigen vector of

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

with the initial approximation eigen vector $[1 \quad 1 \quad 1]^T$. Perform five iterations. **(07 Marks)**

OR

10 a. Find the rank of the matrix

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

by using elementary row transformation. **(06 Marks)**

b. Test the consistency and solve
$x + y + z = -3$, $3x + y - 2z = -2$, $2x + 4y + 2z = 7$ **(07 Marks)**

c. Solve the system of equations $2x - 3y + 20z = 25$, $20x + y - 2z = 17$, $3x + 20y - z = -18$ using Gauss - Seidel method taking (0, 0, 0) as an initial approximate. Carry out 4 iterations. **(07 Marks)**

* * * * *