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Nonlinear Observers for Systems with State Delay and Randomly Missing Measurements

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Abstract

We present a nonlinear observer for continuous time dynamic system with state delay, and randomly missing measurements. Using the Lyapunov energy (LE) functional, we derive sufficient conditions for the local asymptotic stability for the observer state error equations. The observer performance without and with missing measurements is evaluated by simulations implemented in MATLAB. The results validate the theoretical asymptotic behaviour of the proposed nonlinear observer for the system with time delay and randomly missing measurements. We, then extend the nonlinear observer for nonlinear system with state delay and randomly missing measurements in the measurement data level and state vector fusion modes for multisensory data fusion situation. We also, ascertain that the state vector fusion formula for these observers is similar to that for the Kalman/extended Kalman filter. We ascertain that the derived theoretical result automatically extends to these nonlinear observers due to their non-complicated structures.

Keywords: Delayed states, randomly missing data, nonlinear observers, asymptotic results, measurement and state vector level fusion.

INTRODUCTION

In control and estimation-cum-filtering theory, the objective of an observer is to reconstruct the state of a dynamic system using the knowledge of the system input/output data/signals. For linear systems, the system states are generally estimated using the Luenberger observer or Kalman filter. For a nonlinear system one uses an extended Luenberger observer or extended Kalman filter [1].However, there are not many results for state estimation of nonlinear systems with delayed states and randomly missing measurements. There are real life dynamic systems like transportation, chemical reactors, biological systems, computer networks, and communication systems, including wireless sensor networks (WSN), wherein state delays and/or missing measurements could occur; e.g. sometimes, in a data-communications channel a few or many measurements might be missing (from one or more sensors). In such cases, it becomes important to study and evaluate the performance of the data processing algorithms in presence (and despite) of missing measurements. That is, in the absence of certain signals/data during certain time intervals, we would have only the random (measurement) noises present for these channels.

This joint/combined/synergy aspect has not gained much attention in the context of nonlinear systems, though some work has been done in certain special cases [2-10]. In [2-4] some special cases have been considered: a) availability of intermittent observations, b) missing data in online condition monitoring, and c) some packet dropouts. A system with multiple sensor-delay is considered in [5], but the algorithm turns out to be quite involved. The problem of measurement/data outliers and missing data is considered in [6], however, the illustration has been given only for simple time-series case. Although, the refs. [7,8,10] deal with missing observations, however, the aspect of state delay is not treated. In [9] only the problem with system delay is considered. In [11] an EKF-based nonlinear observer has been studied for the system with time-delays and some asymptotic results [11, 12] have been presented, but the case of missing measurements was not studied.

In the present paper, we consider the state delay as well as randomly missing measurements/data in a nonlinear observer in a synergistic manner. Some data may be missing due to: a) a failure of a senor, and/or b) there might be a problem in one or more communication channel/s; in such cases, the received data might be only the noise and the real signal is missing. So, it is very important to handle the situation of missing data in a nonlinear observer in some optimal way. Also, the time delay could be encountered in some real-time systems, due to latency time of certain channels. Thus, the time delay is a key factor that influences the overall system stability and performance; and if these aspects of system's state delay and missing data are not handled appropriately in a tracking algorithm, then we might lose the track.

Hence, we propose a nonlinear observer that would handle state delay as well as randomly missing measurements; and also derive the asymptotic condition for the observer error dynamics. The performance is illustrated with implementation in MATAB. We also, propose nonlinear observer/s for nonlinear continuous time system with state delay and randomly missing measurements in the measurement data level and state vector level fusion modes (MLF, SVF). We also, ascertain that a state vector fusion formula for this observer is similar to that for the Kalman/extended Kalman filter. Many of the presented results and observations (inferences) here, are novel in the area of observer theory and multisensory data fusion.

Nonlinear system and observer error dynamics

Let the nonlinear delayed state model with randomly missing measurements be given as

$$\dot{x}(t) = f(x(t), x(t-\tau), u(t)) \\ y(t) = \gamma H x(t) \quad (1)$$

In (1), often the control input is ignored. Also, the scalar quantity γ is a Bernoulli sequence; in which case we could have written y(k)= γ (k)H(k)x(k), without loss of any generality, and this sequence takes values *0* and *1* randomly; thus, we have $E\{\gamma(k)=1\} = b(k)$ and $E\{\gamma(k)=0\} = 1-b(k)$, with *b* as the percentage of measurements that arrive to/from the sensor node, and E{.} is the mathematical expectation. This, also signifies that some measurement data are randomly missing. The constant *b* is assumed to be known and pre-specified. The initial conditions for the states and delayed state are assumed to be appropriately specified [11]. The variables in (1), have usual appropriate dimensions, and presently we consider that these belong to real 2D space (say H_2 vector spaces). It is also assumed that the nonlinear function f is continuously differentiable. Then, we propose a nonlinear observer for the system of (1) as

$$\dot{\hat{x}}(t) = f(\hat{x}(t), \hat{x}(t-\tau), u(t)) + L(t)(y(t) - \hat{y}(t)) \\ \hat{y}(t) = \gamma H \hat{x}(t) \quad (2)$$

In (2), L(t) is an observer gain matrix of appropriate dimension, and is given as

$$L(t) = P(t) H^T R^{-1} \quad (3)$$

In the case of the observer of (2), R is some positive definite matrix, and P(t) is obtained as the solution of the observer Riccati differential (ORD) equation

$$\dot{P}(t) = P(t) A_0^T(t) + A_0(t) P(t) - b^2 P(t) H^T R^{-1} H P(t) + S + A_1(t) A_1^T(t) \quad (4)$$

We see that the observer gain is obtained from the EKF [11,12]. The required Jacobians for (4) are obtained as

$$A_0(t) = \frac{\partial f(.)}{\partial \hat{x}(t)}; \quad A_1(t) = \frac{\partial f(.)}{\partial \hat{x}(t-\tau)} \quad (5)$$

In (4), we have additional term arising due to the state delay (5). By subtracting (2) from (1) we obtain the following error dynamics

$$\dot{e}(t) = A_0(t)e(t) + A_1(t)e(t-\tau) - L(t)Hb(x(t) - \hat{x}(t)) + \phi(.) \\ = A_0(t)e(t) + A_1(t)e(t-\tau) - bL(t)He(t) + \phi(.) \quad (6)$$

In (6), we have the new nonlinear function as

$$\phi(.) = -A_0(t)e(t) - A_1(t)e(t-\tau) + f(x,t,t-\tau) - f(\hat{x},t,t-\tau) \quad (7)$$

The form of (7) is obtained by adding and subtracting the terms related to the Jacobians in the original equations of the error dynamics. In (7) the full forms for f and ϕ (.) are given for clarity $\phi(.) = \phi(x(t), x(t-\tau), \hat{x}(t), \hat{x}(t-\tau))$ (8)

$$f(x,t,t-\tau) = f(x(t), x(t-\tau)) \\ f(\hat{x},t,t-\tau) = f(\hat{x}(t), \hat{x}(t-\tau))$$

However, for simplicity we will use very compact form of by avoiding the arguments since, these would be implied any way from its defining form (1), and (2), and we have also omitted u(t). We note that the state errors are given as

$$e(t) = x(t) - \hat{x}(t); \quad e(t-\tau) = x(t-\tau) - \hat{x}(t-\tau) \quad (9)$$

Asymptotic stability of observer error dynamics

We consider the following conditions [11,12] for the local asymptotic behavior of the observer error dynamics of (6)

1. The solution of the ORD equation (4) is bounded

$$p_l I \le P(t) \le p_u I \quad (10)$$

with $p_l, p_u >$ 0 as positive constants (since P(t) is also theoretically, positive definite and symmetrical matrix), and are the lower and upper bounds respectively; and I is the identity matrix.

2. The nonlinearity (6)-(8) of the error dynamics is bounded

$$\|\phi(.)\| \le \rho_1 \|x(t) - \hat{x}(t)\|^2 + \rho_2 \|x(t-\tau) - \hat{x}(t-\tau)\|^2 \quad (11)$$

with the bounding constants equal to or greater than zero.

Then, the nonlinear observer error dynamics (6) are locally asymptotically stable, if the conditions 1 and 2 are satisfied.

First, we consider the following normalized LE functional to establish the asymptotic stability of the error dynamics (6)

$$V(t) = e^T(t) Y(t) e(t) \quad (12)$$

In (12), Y(t) is the normalizing/weighting matrix and can be recognized as an information matrix, and is given as Y(t)=P-

^{1}(t).In the case of Kalman filter, P(t) is considered as the covariance matrix of the state error-vector. Here, also it can be considered so, however, since we are not dealing with stochastic noise processes, we can call this matrix as the Gramian matrix, and still retain the name 'information matrix' for the inverse of matrix P(t). In such a case the variables x(.), and y(.) can be considered as the generalized 'random' variables. We can see that LE functional is positive definite because it is governed by the condition 1, the inequality of (10) as follows

$$\frac{1}{p_u}\|e(t)\|^2 \le e^T(t)Y(t)e(t) \le \frac{1}{p_l}\|e(t)\|^2 \tag{13}$$

The idea in obtaining the asymptotic result is that the time derivative of the LE functional, (12), under the constraints governed by error dynamics (6), and (3) and (4), should be negative definite. Hence, we obtain this time derivative as

$$\begin{aligned}\dot{V}(t) &= e^T(t)\dot{Y}(t)e(t) + e^T(t)\{A_0^T(t)Y(t) + Y(t)A_0(t) \\ &\quad - bH^T L^T(t)Y(t) - bY(t)L(t)H\}e(t) + e^T(t-\tau)A_1^T(t)Y(t)e(t) \\ &\quad + e^T(t)Y(t)A_1(t)e(t-\tau) \\ &\quad + 2e^T(t)Y(t)\phi(.).\end{aligned} \tag{15}$$

Next, we substitute for $\dot{Y}(t) = -Y(t)\dot{P}(t)Y(t)$, and (3) and (4) in (15) to obtain

$$\begin{aligned}\dot{V}(t) &= -e^T(t)Y(t)A_1(t)A_1^T(t)Y(t)e(t) + e^T(t-\tau)A_1^T(t)Y(t)e(t) + e^T(t)Y(t)A_1(t)e(t-\tau) \\ &\quad - e^T(t-\tau)e(t-\tau) + e^T(t-\tau)e(t-\tau) - e^T(t)Y(t)SY(t)e(t) \\ &\quad - b^2e^T(t)H^T R^{-1}He(t) + 2e^T(t)Y(t)\phi(.).\end{aligned} \tag{16}$$

In obtaining (16), because of the substitution of (3), and (4), several common terms cancel out. In (16), we have added and subtracted the term $e^T(t-\tau)e(t-\tau)$, and thus due to the structure of the first four terms of (16), we can combine these in the compact form $-\|A_1^T(t)Y(t)e(t) - e(t-\tau)\|^2$, and by using this in (16) we obtain

$$\begin{aligned}\dot{V}(t) &= -\|A_1^T(t)Y(t)e(t) - e(t-\tau)\|^2 - e^T(t)Y(t)SY(t)e(t) \\ &\quad - b^2e^T(t)H^T R^{-1}He(t) + e^T(t-\tau)e(t-\tau) + 2e^T(t)Y(t)\phi(.).\end{aligned} \tag{17}$$

We can then get the following equivalent inequality by dropping the first compact term (this does not affect the inequality) from (17)

$$\dot{V}(t) \le -e^T(t)Y(t)SY(t)e(t) - b^2e^T(t)H^T R^{-1}He(t) + e^T(t-\tau)e(t-\tau) + 2e^T(t)Y(t)\phi(.). \tag{18}$$

In (18), we assume that $\|R^{-1}\| \le 1/r; \|H^T H\| \le h^2$, (r and h being positive constants) and $\|e(t)\|^2 \le \varepsilon^2$ and since, these are known and pre-specified quantities, or should be finite, we obtain

$$\dot{V}(t) \le -e^T(t)Y(t)SY(t)e(t) - \frac{h^2\varepsilon^2 b^2}{r} + \|e(t-\tau)\|^2 + 2e^T(t)Y(t)\phi(.). \tag{19}$$

Then, using the inequality from (11), we obtain

$$\dot{V}(t) \le -e^T(t)Y(t)SY(t)e(t) + 2e^T(t)Y(t)\{\rho_1\|e(t)\|^2 + \rho_2\|e(t-\tau\|^2\} - \frac{h^2b^2}{r}\varepsilon^2 + \|e(t-\tau\|^2 \tag{20}$$

$$\dot{V}(t) \le -e^T(t)Y(t)SY(t)e(t) + 2\{\frac{\rho_1}{p_u}\|e(t)\|\|e(t)\|^2 + \frac{\rho_2}{p_u}\|e(t)\|\|e(t-\tau\|^2\} - \frac{h^2b^2}{r}\varepsilon^2 + \|e(t-\tau\|^2$$

$$\dot{V}(t) \le -\frac{s_l}{p_u^2}\|e(t)\|^2 + 2\{\frac{\rho_1}{p_u}\|e(t)\|\|e(t)\|^2 + \frac{\rho_2}{p_u}\|e(t)\|\|e(t-\tau\|^2\} - \frac{h^2b^2}{r}\varepsilon^2 + \|e(t-\tau\|^2 \tag{21}$$

$$\dot{V}(t) \le -\{\frac{s_l}{p_u^2} - \frac{2\rho_1}{p_u}\|e(t)\|\}\|e(t)\|^2 - \{\frac{h^2b^2}{r} - 1 - \frac{2\rho_2}{p_u}\|e(t)\|\}\|e(t-\tau\|^2 \tag{22}$$

In (21), s_l is the smallest (positive) eigenvalue of the matrix S, that is positive definite. For $\|e(t)\| \le \varepsilon$, we have the following condition from (22)

$$\dot{V}(t) \le -\{\frac{s_l}{p_u^2} - \frac{2\rho_1}{p_u}\varepsilon\}\|e(t)\|^2 - \{\frac{h^2b^2}{r} - 1 - \frac{2\rho_2}{p_u}\varepsilon\}\|e(t-\tau\|^2 \tag{23}$$

$$\dot{V}(t) \le -\{\frac{s_l}{2\rho_1 p_u}; \frac{(h^2b^2 - r)p_u}{2\rho_2 r}\}\|e(t)\|^2 \tag{24}$$

Since, all the constants and bounds appearing in the {.;.} in (24) are positive, and also, we regard the bounding constants in (11), as positive, (in the case that these are really equal to zero, we can assign them slightly positive values, without loss of any generality, since these constants are arbitrary, and we can always assure that $h^2b^2 > r$), we see that the time derivative of the Lyapunov energy functional is bounded from above as in (24). Since, the Lyapunov energy functional is positive definite as in (12), and its time derivative is locally negative definite as in (24), the observer error dynamics of the newly proposed nonlinear observer for system with state delay, and randomly missing measurements/data is locally asymptotically stable.

Nonlinear observer for sensor data fusion

Multisensory data fusion is an evolving technology at software/algorithms and hardware levels and is defined as: an act, that could be additive, multiplicative, operative, and/or logical by which a) the quantitative information in the sense of Fisher's information matrix is enhanced by fusing/combining data from more than one sensor or source, and/or b) the prediction accuracy is enhanced, compared to the usage of a single sensor-data or source [13].

Observer for the measurements/data level fusion

Here, we consider the delayed state model, and missing measurements/data in the measurement data level fusion. The state model is the same as (1), and the measurement model for the two-sensor scheme is given as

$$\begin{aligned}\dot{x}(t) &= f(x(t), x(t-\tau), u(t)) \\ y^i(t) &= \gamma^i H^i x(t)\end{aligned} \tag{25}$$

In (25), $y^i(t)$, i=1,2 is the output from the first (i=1), and the second (i=2) sensors. Here also, the scalar quantity γ^i is a Bernoulli sequence; in which case we could have written y(k)= γ^i (k)H(k)x(k), without loss of any generality, and this sequence takes values *0* and *1* randomly; thus, we have $E\{\gamma^i(k)=1\}=b^i(k)$ and $E\{\gamma^i(k)=0\}=1-b^i(k)$, with b^i as the percentage of measurements that arrive to/from the sensor node. This, also signifies that some measurement data are randomly missing. The constant b^i are assumed to be known and pre-specified. A nonlinear observer for the system of (25) is specified as

$$\begin{aligned}\dot{\hat{x}}_f(t) &= f(\hat{x}_f(t),\hat{x}_f(t-\tau),u(t)) + L(t)(y(t)-\hat{y}(t))\\ \hat{y}(t) &= \gamma H\hat{x}_f(t)\end{aligned} \tag{26}$$

The 'f' denotes the fused state obtained as a result of the MLF of the two measurements from the two sensors, combined at the data level, and (26) is rewritten as

$$\begin{aligned}\dot{\hat{x}}_f(t) &= f(\hat{x}_f(t),\hat{x}_f(t-\tau),u(t)) + L_c(t)(y_{cm}(t)-bH_c\hat{x}_f(t))\\ \hat{y}_c(t) &= bH_c\hat{x}_f(t)\end{aligned} \tag{27}$$

In (27), y_{cm} and H_c are appropriate composite vectors/matrices to account for the direct data level fusion of the measurements/data coming from the two sensors, and b^i can be appropriately accounted for. In (27), $L_c(t)$ is the observer gain matrix of appropriate dimension, and is given as

$$L_c(t) = P(t)H_c^{\,T}R_c^{-1} \tag{28}$$

In case of the observer gain, (28), R_c is some positive definite matrix reflecting the combination of the measurements/data, and P(t) is obtained as the solution of the ORD equation

$$\dot{P}(t) = P(t)A_0^T(t) + A_0(t)P(t) - b^2P(t)H_c^{\,T}R^{-1}H_cP(t) + S + A_1(t)A_1^T(t) \tag{29}$$

By subtracting (27) from (26) we obtain the following error dynamics

$$\begin{aligned}\dot{e}_f(t) &= A_0(t)e_f(t) + A_1(t)e_f(t-\tau) - L_c(t)H_cb(x_f(t)-\hat{x}_f(t)) + \phi_f(.)\\ &= A_0(t)e_f(t) + A_1(t)e_f(t-\tau) - bL_c(t)H_ce_f(t) + \phi_f(.)\end{aligned} \tag{30}$$

In (30), 'f' denotes the fused condition, and it does not have any effect on the dimension of the state vector and hence state error, it only reflects the fact that the observer state has the combined effect of two measurements. In (30), we have the new nonlinear function as

$$\phi_f(.) = -A_0(t)e_f(t) - A_1(t)e_f(t-\tau) + f(x_f,t,t-\tau) - f(\hat{x}_f,t,t-\tau) \tag{31}$$

In (31), the full forms for nonlinear functions and $\phi_f(.)$ are given for clarity as

$$\begin{aligned}\phi_f(.) &= \phi(x_f(t),x_f(t-\tau),\hat{x}_f(t),\hat{x}_f(t-\tau))\\ f(x_f,t,t-\tau) &= f(x_f(t),x_f(t-\tau))\\ f(\hat{x}_f,t,t-\tau) &= f(\hat{x}_f(t),\hat{x}_f(t-\tau))\end{aligned} \tag{32}$$

We note that the state errors are given as

$$e_f(t) = x_f(t)-\hat{x}_f(t);\;\; e_f(t-\tau) = x_f(t-\tau)-\hat{x}_f(t-\tau) \tag{33}$$

We observe from (27)-(30), and the conditions of (10), and (11), that the theoretical development of Section 3 is equally applicable to the observer error dynamics of (30), and hence, we ascertain by induction that observer error dynamics of the newly proposed nonlinear observer (27) for systems with state delay, and randomly missing measurements/data in the data level fusion is locally asymptotically stable.

Observer for the state vector fusion

In the state vector level fusion (SVF), we consider that the measurements coming from two sensors are individually processed at each local sensor node, and then the estimated state vector is obtained from the individual estimate by the SVF formula. We consider the nonlinear dynamics model as in (25)

$$\begin{aligned}\dot{x}^i(t) &= f^i(x^i(t),x^i(t-\tau))\\ y^i(t) &= \gamma^iH^ix^i(t)\end{aligned} \tag{34}$$

In (34), i=1,2 are the two sensors, the measurement data from which are processed by an individual observer at each sensor node as is done in Section 3.

$$\begin{aligned}\dot{\hat{x}}^i(t) &= f^i(\hat{x}^i(t),\hat{x}^i(t-\tau)) + L^i(t)(y^i(t)-\hat{y}^i(t))\\ \hat{y}^i(t) &= \gamma^iH^i\hat{x}^i(t)\end{aligned} \tag{35}$$

Once, the state estimates are obtained at each sensor node (of course concurrently by two processors), we can fuse these estimates by using the following formula as is done in the case of KF

$$\hat{x}_f = \hat{x}^1 + \hat{P}^1\left(\hat{P}^1+\hat{P}^2\right)^{-1}\left(\hat{x}^2-\hat{x}^1\right) \tag{36}$$

$$\hat{P}_f = \hat{P}^1 - \hat{P}^1\left(\hat{P}^1+\hat{P}^2\right)^{-1}\hat{P}^{1^T} \tag{37}$$

In (36) we have the individual state estimate obtained from the corresponding observer (2), that has processed the measurements from the corresponding sensor (i=1,2), and in (37), we have the Gramians(P for i=1,2), obtained by solving the corresponding matrix ORD equation (4). So, we look upon P(.), i=1,2; as the weighting matrices used in the fusion rule (36). Although, in the case of KF/EKF, P(.) are considered as the covariance matrices; so far as the SVF is concerned, these are the appropriate weighting factors obtained from the covariance matrices, and incidentally happened to be the covariance matrices themselves. In the case of the observers, we can consider these weighting factors as obtained from the Gramians (representing some uncertainty or dispersion of the estimate from the true value), and just happen to be the Gramians themselves. Again, we ascertain that the theoretical development of Section 3 is equally applicable to the observers of (35), since each is an individual observer as in (2), and hence, by induction observer error dynamics of the systems with state delay, and randomly missing measurements/data for the state vector fusion are locally asymptotically stable.

Evaluation of the nonlinear observer

The performance of the presented observer is validated using numerical simulations carried out in MATLAB. The simulations are done for a period of 4 seconds with a sampling interval of 0.01 sec. We consider the following nonlinear dynamic system

$$\begin{aligned}\dot{x}_1(t) &= -(x_1(t)+3.3)(x_1(t)+x_2(t))\\ \dot{x}_2(t) &= -10x_2(t-\tau)+10x_2(t)-(3x_2(t-\tau)-10)x_1(t-\tau)\end{aligned} \quad (34)$$

y(t)=x_1(t)

The model in (34) is the prey, x1(t) – predator, x2(t) population dynamics model [11]. For, data simulation as well as observer states, the dynamic equations (1), and (2) are solved by using Euler integration method, and hence, these equations and the Jacobians are properly represented in the discrete-form as

$$\begin{aligned}x_1(k) &= (-x_1(k-1)-3.3)(x_1(k-1)+x_2(k-1))\\ x_2(k) &= -10x_2(k-2)+10x_2(k-1)-3x_2(k-2)x_1(k-2)+10x_1(k-2)\end{aligned} \quad (35)$$

$$A_0(k)=\begin{bmatrix}-2x_1(k-1)-3.3-x_2(k-1) & -x_1(k-1)-3.3\\ 0 & 10\end{bmatrix} \quad (36)$$

$$A_1(k)=\begin{bmatrix}0 & 0\\ -3x_2(k-2)+10 & -10-3x_1(k-2)\end{bmatrix} \quad (37)$$

The value of R (or 'r') appearing in (4) is used as a tuning parameter. The state initial conditions used for the simulation and the nonlinear observer are chosen appropriately. In order to implement the observer algorithm, we need to solve the matrix ORD equation (4), and for this we use the following transformation [1]

$$a = P(t)\,d \quad (38)$$

to obtain the following differential equations, appropriately from (4)

$$\dot{d} = -A_0^T d + b^2 H^T R^{-1} H\, a \quad (39)$$

$$\dot{a} = (I + A_1 A_1^T)\mathrm{d} + A_0 a \quad (40)$$

The equations (39) and (40) are solved by using the transition matrix method [1], then using (38) we get P(t). The performance of the observer with no missing measurements/data is illustrated in Figure 1. Figure 2 depicts the convergence of the eigenvalues of the matrix P (left graph), and the true and predicted measurements. Next, the measurement data are missed at level some level randomly. The performance of the observer with missing measurements/data is illustrated in Figure 3. Figure 4 depicts the convergence of the eigenvalues of the matrix P (left graph), and the true and predicted measurements when the measurement data are missing at some level.

From Figures1 and 3, it is clear that the proposed nonlinear observer is asymptotically stable and satisfies the condition (24), when b=1 (no missing data), and when b=some value (some data are missing).

Concluding remarks

Some new results on nonlinear observer for system with state delay and randomly missing measurements have been obtained. The performance of such an observer has been validated using prey-predator population dynamics nonlinear model simulated in MATLAB. The asymptotic stability result obtained for such an observer using the Lyapunov energy functional has been validated by the behaviour of the eigenvalues of the Gramian matrices. The performance of the observer with and without missing measurement data has been found to be very satisfactory and the results corroborate the theoretical result presented in Section3. Also, the structures of the nonlinear observers for the system with state delay and randomly missing data have been presented, for measurement level fusion and state vector fusion, in comparison with the conventional filtering algorithms used for data fusion, and again for these observer based fusion schemes the same theoretical results hold true.

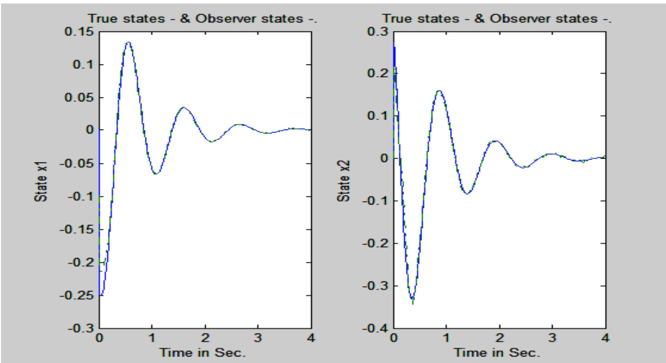

Figure 1: Time history match of the true (-) and observer states (-.); no missing data.

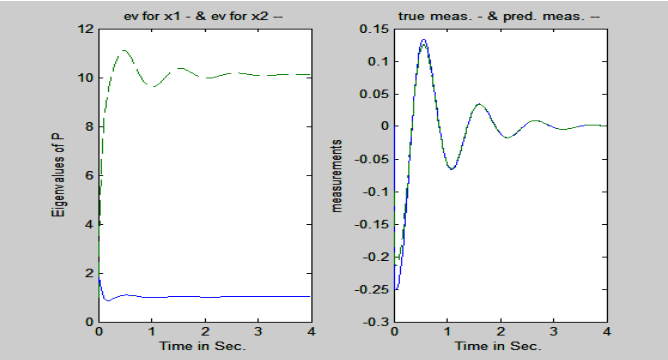

Figure 2: Eigenvalues of the P showing convergence/satisfaction of the condition of (10),related to state x1 (-), & state x2 (--); and the true (-), and predicted measurements (--); No missing data.

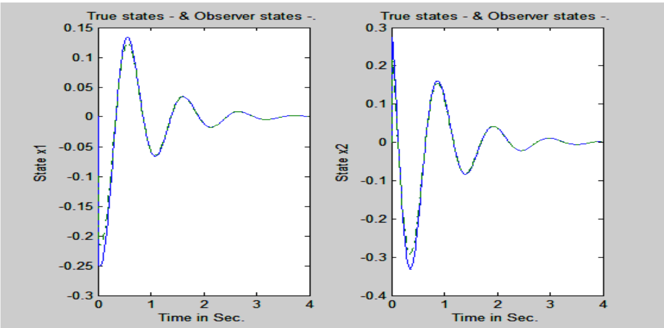

Figure 3: Time history match of the true (-) & observer (-.) states; some missing data.

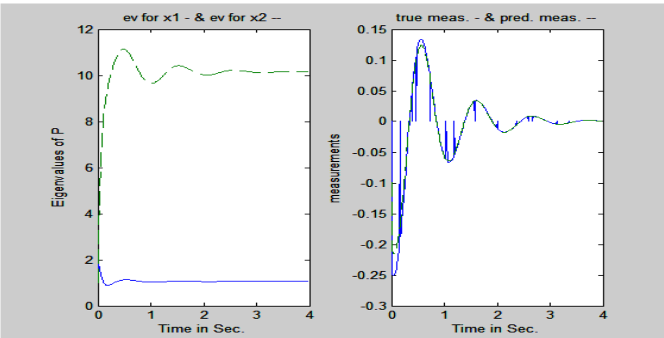

Figure 4: Eigenvalues of the P showing convergence/satisfaction of the condition of (10), related to state x1 (-), & state x2 (--); and the true (-), and predicted measurements (--); some missing data.

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