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18AI56

Fifth Semester B.E. Degree Examination, June/July 2025
Mathematics for Machine Learning

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Define vector space. Write the polynomial $f(t) = at^2 + bt + c$ as a linear combination of the polynomials $P_1 = (t-1)^2$, $P_2 = t - 1$ and $P_3 = 1$. **(06 Marks)**

b. Suppose $u = (1, -3, 4)$ and $v = (3, 4, 7)$. Find
(i) $d(u, v)$, the distance between the vectors u and v
(ii) The angle between the vectors u and v
(iii) The projection of u and v **(06 Marks)**

c. Compute the following matrix products, if possible.

(i) $\begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ **(08 Marks)**

OR

2 a. Solve using Gaussian Elimination, all solutions of the inhomogeneous equation system $Ax = b$ with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$ **(10 Marks)**

b. Which of the following sets are subspace of R^3?
i) $A = \{(\lambda, \lambda + \mu_3, \lambda - \mu^3) \mid \lambda, \mu \in R\}$
ii) $B = \{(\lambda^2, -\lambda^2, 0) \mid \lambda \in R\}$
iii) Let γ be in R
$C = \{(\varepsilon_1, \varepsilon_2, \varepsilon_3) \in R^3 \mid \varepsilon_1 - 2\varepsilon_2 + 3\varepsilon_3 = \gamma\}$
iv) $D = \{(\varepsilon_1, \varepsilon_2, \varepsilon_3) \in R^3 \mid \varepsilon_2 \in Z\}$ **(10 Marks)**

Module-2

3 a. Find the QR factorization of

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$ **(06 Marks)**

b. Compute det A where

$$A = \begin{bmatrix} 3 & -7 & 8 & 9 & -6 \\ 0 & 2 & -5 & 7 & 3 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 2 & 4 & -1 \\ 0 & 0 & 0 & -2 & 0 \end{bmatrix}$$

(06 Marks)

c. Let W be the subspace of R^4 orthogonal to $u_1 = (1, 1, 2, 2)$ and $u_2 = (0, 1, 2, -1)$. Find (i) an orthogonal basis of W (ii) an orthonormal basis of W. (08 Marks)

OR

4 a. Find the eigen value and eigen vectors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

(06 Marks)

b. Find the matrix P which transform the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$ to the diagonal form.

(06 Marks)

c. Find the singular value decomposition of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

(08 Marks)

Module-3

5 a. Consider the following functions :

$f_1(x) = \sin(x_1)\cos(x_2)$, $x \in R^2$

$f_2(x, y) = x^T y$, $x, y \in R^n$

$f_3(x) = xx^T$, $x \in R^n$

i) Compute the Jacobian.

ii) What are the dimensions of $\frac{\partial f_i}{\partial x}$ (10 Marks)

b. Compute the Taylor polynomial T_n, $n = 0 \ldots.5$ of $f(x) = \sin(x) + \cos(x)$ at $x_0 = 0$ (10 Marks)

OR

6 a. Compute the derivative of the function $h(x) = (2x + 1)^4$ using Chain Rule. (06 Marks)

b. Find the value of the constraints λ and μ such that the surface $\lambda x^2 - \mu yz = (\lambda + 2)$ and $4x^2y + z^3 = 4$ intersect orthogonally at the point (1, –1, 2). (08 Marks)

c. Given $g(z, v) := \log p(x, z) - \log q(z, v)$

$z := t(\varepsilon, v)$

for differentiable functions p, q, t.

Using chain rule compute the gradient $\frac{d}{dv} g(z, v)$ (06 Marks)

Module-4

7 a. Consider the following bivariate distribution P(x, y) of two discrete random variables x and y.

Y					
y_1	0.01	0.02	0.03	0.1	0.1
y_2	0.05	0.1	0.05	0.07	0.2
y_3	0.1	0.05	0.03	0.05	0.04
X	x_1	x_2	x_3	x_4	x_5

Compute (i) Marginal distribution P(x) and P(y)
(ii) Conditional distribution $P(x \mid Y = y_1)$ and $P(y \mid X = x_3)$ **(08 Marks)**

b. State and prove the Baye's theorem. **(06 Marks)**

c. Let A and B be two events, not mutually exclusives connected with a random experiment E. if P(A) =1/4 , P(B) = 2/5 and $P(A \cup B) = 1/2$. Find the values of the following probabilities
(i) $P(A \cap B)$ (ii) $P(A \cap B')$ (iii) $P(A \cup B')$ **(06 Marks)**

OR

8 a. Determine the binomial distribution for which Mean = 2 × variance and mean + variance = 3 **(06 Marks)**

b. The probability distribution of a random variable x is given below:

X	-2	-1	0	1	2
P(x)	0.2	0.1	0.3	0.3	0.1

Find (i) E(x) (ii) V(x) (iii) E(2x – 3) (iv) V(2x – 3) **(06 Marks)**

c. There is an outbreak of an epidemic in a particular city and government has decided to vaccinate the people residing in that city as a precautionary measure. Now it is observed that, while vaccinating the people, the probability that an individual suffers due to a bad reaction from a certain injection is 0.001. Determine the probability that out of 2000 individuals
(i) Exactly 3 people will suffer due to a bad reaction.
(ii) More than 2 people will suffer due to a bad reaction.
(iii) No one will suffer due to a bad reaction. **(08 Marks)**

Module-5

9 a. What is Gradient Descent? With the help of gradient descent find the point at which the function $f(x_1, x_2) = x_1^2 - 2x_1x_2 + 2x_2^2 + 2x_1$ has optimal solution. **(06 Marks)**

b. Consider the univariate function $f(x) = x^3 + 6x^2 - 3x - 5$. Find its stationary points and indicate whether they are maximum, minimum or saddle points. **(06 Marks)**

c. Consider whether the following statements are true or false:
i) The sum of any two convex functions is convex.
ii) The difference of any two convex functions is convex.
iii) The product of any two convex functions is convex.
iv) The maximum of any two convex functions is convex. **(08 Marks)**

OR

10 a. Using Lagrange's multiplier, find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface area is 432 sq.cm. **(06 Marks)**

b. Prove that non-negative weighted sum of convex functions is always convex. **(06 Marks)**

c. Consider the following convex optimization problem

$$\min_{w \in R^D} \frac{1}{2} w \ \ w$$

Subject to $w \ \ x \geq 1$

Derive the Lagrangian dual by introducing the Lagrange multiplier λ. **(08 Marks)**

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