



First Semester MCA Degree Examination, Dec.2024/Jan.2025

Discrete Mathematics and Graph Theory

Time: 3 hrs.

Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

Module - 1			M	L	C
Q.1	a.	If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 7\}$, $B = \{4, 5, 6, 7\}$ and $C = \{1, 3, 6\}$. Compute the following: (i) $\overline{A \cup C}$ (ii) $\overline{A} \cap \overline{B}$ (iii) $A \cap B \cap C$ (iv) $B - A$ (v) $A - B$	6	L1	CO1
	b.	Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{6, 7, 8, 9, 10\}$ and $f: A \rightarrow B$ be a function defined by $f = \{(1,7)(2,7)(3,8)(4,6)(5,9)(6,9)\}$. Determine $f^{-1}(6)$ and $f^{-1}(9)$. Also if $B_1 = \{7, 8\}$, $B_2 = \{8, 9, 10\}$ then find $f^{-1}(B_1)$ and $f^{-1}(B_2)$.	7	L2	CO1
	c.	Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$.	7	L2	CO1
OR					
Q.2	a.	For any two sets A and B , prove the Demorgan's laws.	6	L1	CO1
	b.	State pigeon-hole principle. Show that if 50 books in a library contain a total of 27551 pages, one of the books must have atleast 552 pages.	7	L2	CO1
	c.	In a class of 52 students, 30 are studying C++, 28 are studying pascal and 13 are studying both languages. How many in this class are studying at least one of these languages? How many are studying neither of these languages?	7	L2	CO1
Module - 2					
Q.3	a.	Define tautology. Show that $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is a tautology	7	L2	CO2
	b.	Write the converse, inverse and the contra positive of the conditional statement: "If oxygen is a gas then Gold is compound".	6	L2	CO2
	c.	Prove the following is valid argument : $\begin{array}{l} p \rightarrow r \\ \sim p \rightarrow q \\ \hline q \rightarrow s \\ \hline \therefore \sim r \rightarrow s \end{array}$	7	L2	CO2
OR					
Q.4	a.	Prove the following using the laws of logic: $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$	7	L2	CO2
	b.	Negate and simplify: (i) $\forall x, [p(x) \wedge \sim q(x)]$. $\exists x, [(p(x) \vee q(x)) \rightarrow r(x)]$.	6	L2	CO2
	c.	Give the direct proof of the following statement "If n is an odd integer, then n^2 is odd."	7	L2	CO2
Module - 3					
Q.5	a.	Define graph and explain the types of graph.	8	L1	CO3

	b.	Prove that the number of vertices of odd degree in a graph is always even.	6	L2	CO3
	c.	Define isomorphic graph and verify the following graphs are isomorphic or not.	6	L2	CO3
OR					
Q.6	a.	Explain the following graphs: (i) Bi- partite graph (ii) Sub graphs (iii) Walk (iv) Path	10	L1	CO3
	b.	Prove that a simple graph with n vertices and K components can have at most $(n-k)(n-k+1)/2$ edges.	10	L2	CO3
Module – 4					
Q.7	a.	State and prove necessary condition of a graph to be a Euler graph.	10	L2	CO4
	b.	List and explain the different operations on graph.	10	L2	CO4
OR					
Q.8	a.	Define digraph. Find the indegree and outdegree of the following graph:	8	L2	CO4
	b.	Illustrate the travelling salesman problem using a graph.	6	L2	CO4
	c.	List and explain different digraphs and binary relations.	6	L2	CO4
Module – 5					
Q.9	a.	Prove that every tree with two or more vertices is 2- Chromatic	10	L2	CO5
	b.	Explain the following for chromatic polynomial: (i) Finding a maximal independent set. (ii) Finding all maximal independent set.	10	L2	CO5
OR					
Q.10	a.	Prove that the vertices of every planar graph can be properly colored with five colors.	10	L2	CO5
	b.	Explain the Greedy coloring algorithm.	10	L2	CO5
