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Third Semester B.E. Degree Examination, Dec.2024/Jan.2025
Additional Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the modulus and amplitude of $\frac{(3 - \sqrt{2}i)^2}{1 + 2i}$ (07 Marks)
- b. Show that $\left(\frac{\cos \theta + i \sin \theta}{\sin \theta + i \cos \theta}\right)^4 = \cos 8\theta + i \sin 8\theta$ (06 Marks)
- c. If $\vec{a} = 3i - 2j + 4k$ and $\vec{b} = i + j - 2k$ find the angle between vector \vec{a} and \vec{b} (07 Marks)

OR

- 2 a. Find the value of λ such that the vectors $\vec{a} = 2i - 3j + k$, $\vec{b} = i + 2j - 3k$, $\vec{c} = j + \lambda k$ are co-planar. (07 Marks)
- b. Find the real part of $\frac{1}{(1 + \cos \theta) + i \sin \theta}$ (06 Marks)
- c. If $\vec{A} = i - 2j - 3k$, $\vec{B} = 2i + j - k$, $\vec{C} = i + 3j - k$ find,
 i) $(\vec{A} \times \vec{B}) \times (\vec{B} \times \vec{C})$ ii) $(\vec{A} \times \vec{B}) \cdot \vec{C}$ (07 Marks)

Module-2

- 3 a. Find the nth derivative of $\sin^3 x \cos^2 x$. (07 Marks)
- b. If $Y = a \cos(\log x) + b \sin(\log x)$, find the value of $x^2 y_{n+2} + (2n + 1)x y_{n+1} + (n^2 + 1)y_n = 0$ (06 Marks)
- c. Find the angel between the curves $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$. (07 Marks)

OR

- 4 a. Obtain by Maclaurins theorem the first four terms of $\log \sec x$. (07 Marks)
- b. If $u = f(y-z, z-x, x-y)$ then find the value of $u_x + u_y + u_z$. (06 Marks)
- c. If $u = \frac{yz}{x}$, $v = \frac{xz}{y}$, $w = \frac{xy}{z}$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ (07 Marks)

Module-3

- 5 a. Obtain the reduction formula for $\int \sin^n x \, dx$ and hence find $\int \sin^5 x \, dx$ (07 Marks)
- b. Evaluate $\int_0^{\infty} \frac{x^2}{(1+x^2)^{7/2}} dx$ (06 Marks)
- c. Evaluate $\int_1^2 \int_3^4 (xy + e^y) dy dx$ (07 Marks)
- OR**
- 6 a. Evaluate $\int_0^{\pi/6} \cos^4 3\theta \sin^3 6\theta d\theta$ (07 Marks)
- b. Evaluate $\int_{1/x}^3 \int_{1/x}^1 \int_0^{\sqrt{xy}} xyz \, dz \, dy \, dx$ (06 Marks)
- c. Evaluate $\int_0^{2a} x^2 \sqrt{2ax - x^2} \, dx$ (07 Marks)

Module-4

- 7 a. A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$, $z = 2t - 5$, find the components of velocity and acceleration at $t = 1$ along the direction $2i + j + 2k$. (07 Marks)
- b. Find the directional derivative of $x^2 y z^3$ at $(1, 1, 1)$ in the direction of $i + j + 2k$. (06 Marks)
- c. If $\vec{F} = \text{Grad}(x^3 y + y^3 z + z^3 x - x^2 y^2 z^2)$ then find $\text{div } \vec{F}$ at $(1, 1, 1)$ (07 Marks)
- OR**
- 8 a. If $\vec{V} = 3xy^2 z^3 i + y^3 z^2 j - 2y^2 z^3 k$ and $\vec{F} = (x^2 y z) i + (y^2 - z x) j + (z^2 - x y) k$ then prove that \vec{V} is solenoidal and \vec{F} is irrotational. (07 Marks)
- b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, 1, -2)$. (06 Marks)
- c. Find the constants a, b, c such that $\vec{F} = (x + y + az) i + (bx + 2y - z) j + (x + cy + 2z) k$ is irrotational also find ϕ such that $\vec{F} = \nabla \phi$. (07 Marks)

Module-5

- 9 a. Solve the differential equation $e^{-y} \sec^2 y \, dy = dx + x \, dy$ (07 Marks)
- b. Solve $(y^2 + 2xy) dx + (2x^2 + 3xy) dy = 0$ (06 Marks)
- c. Solve $(1 + x^3) \frac{dy}{dx} + 6x^2 y = e^x$ (07 Marks)
- OR**
- 10 a. Solve $y(2x - y + 1) dx + x(3x - 4y + 3) dy = 0$ (07 Marks)
- b. Solve $e^y \left(\frac{dy}{dx} + 1 \right) = e^x$ (06 Marks)
- c. Solve $xy(1 + xy^2) \frac{dy}{dx} = 1$ (07 Marks)