CBCS SCHEME

USN

21MAT41

Fourth Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025
Complex Analysis, Probability and Statistical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. State and prove Cauchy – Riemann equations in Cartesian form. **(06 Marks)**

b. Show that $w = f(z) = z + e^z$ is analytic and hence obtain its derivatives. **(07 Marks)**

c. Evaluate $\int_c |z|^2 dz$ where c is the square with vertices (0, 1), (1, 0), (1, 1), (0,1) **(07 Marks)**

OR

2 a. Find the analytical function f(z) whose real part is $u = e^{2x}(x \cos 2y - y \sin 2y)$. **(06 Marks)**

b. State and prove Cauchy's integral formula. **(07 Marks)**

c. Evaluate $\int_c \frac{dz}{z^2 - 4}$ where $c : |z| = 3$ **(07 Marks)**

Module-2

3 a. Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ **(06 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$, then prove that

$$\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = 0$$ **(07 Marks)**

c. Express $x^3 + 2x^2 - x - 1$ in terms of Legendre polynomials. **(07 Marks)**

OR

4 a. Show that $J_{-n}(x) = (-1)^n J_n(x)$ where n is a positive integer. **(06 Marks)**

b. Obtain the series solution of the Legendre's equation

$$(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

leading to Legendre polynomial of order n. **(07 Marks)**

c. Express $4x^3 - 2x^2 - 3x + 8$ in terms of Legendre polynomial. **(07 Marks)**

Module-3

5 a. Find Karl Pearson's coefficient of correlation

x :	1	2	3	4	5	6	7
y :	9	8	10	12	11	13	14

(06 Marks)

b. Fit a straight line $y = ax + b$ for the data

x :	50	70	100	120
y :	12	15	21	25

(07 Marks)

c. Fit a second degree parabola $y = ax^2 + bx + c$ for the data

x :	1	2	3	4	5
y :	10	12	13	16	19

(07 Marks)

OR

6 a. Ten students got the following percentage of marks in two subjects x and y. Compute their rank correlation coefficient.

Marks in x	78	36	98	25	75	82	90	62	65	39
Marks in y	84	51	91	60	68	62	86	58	53	47

(06 Marks)

b. Obtain the lines of regression and hence find the coefficient of correlation for the data:

x :	1	2	3	4	5
y :	2	5	3	8	7

(07 Marks)

c. Using the method of square, fit a curve $y = ax^b$ for the following data:

x :	1	2	3	4	5
y :	0.5	2	4.5	8	12.5

(07 Marks)

Module-4

7 a. A random variable X has the following probability function:

X :	-3	-2	-1	0	1	2	3
P(X) :	K	2K	3K	4K	3K	2K	K

Find K. Also find $P(X \le 1)$, $P(X > 1)$, $P(-1 < X \le 2)$ **(06 Marks)**

b. Derive the mean and variance of Poisson distribution. **(07 Marks)**

c. In 800 families with 5 children each how many families would be expected to have (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys (iv) atmost 2 girls by assuming probabilities for boys and girls to be equal. **(07 Marks)**

OR

8 a. A random variable X has density function

$$f(x) = \begin{cases} Kx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Find K. Also find (i) $P(1 < X < 2)$ (ii) $P(X \le 1)$ (iii) $P(X > 1)$ (iv) Mean (v) Variance **(06 Marks)**

b. The number of telephonic lines busy at an instant line is a binomial variate with a probability 0.1. If 10 lines are chosen at random, what is the probability that (i) no line is busy (ii) all lines are busy and (iii) at least one line is busy (iv) at most 2 lines are busy. **(07 Marks)**

c. The marks of 1000 students in an examination follows the normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 (iii) between 65 and 75. **(07 Marks)**

Module-5

9 a. The joint probability distribution for two random variables X and Y is as follows:

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Compute (i) E(X) and E(Y) (ii) E(XY) (iii) σ_X and σ_Y **(06 Marks)**

b. Define (i) Null hypothesis (ii) Type -I and Type -II errors (iii) Level of significance (iv) Degrees of freedom. **(07 Marks)**

c. A sample of 100 tyres is taken from a lot. The mean life of tyres is found to be 40,650 kms with a standard deviation of 3260. Can it be considered as a true random sample from a population with mean life of 40,000 kms (use 0.05 level of significance). (Given $z_{0.05} = 1.96$, $z_{0.01} = 2.58$). **(07 Marks)**

OR

10 a. Determine (i) Marginal distribution (ii) Covariance between the variables X and Y. if the joint probability distribution is

X \ Y	3	4	5
2	1/6	1/6	1/6
5	1/12	1/12	1/12
7	1/12	1/12	1/12

(06 Marks)

b. A certain stimulus administered of the 12 patients resulted in the following change in the blood pressure. 5, 2, 8, –1, 3, 0, 6, –2, 1, 5, 0, 4. Can it be concluded that stimulus will increase the blood pressure (Note: $t_{0.05}$ for 11 d.f is 2.201). **(07 Marks)**

c. Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches. ($t_{0.05} = 2.262$ for 9 d.f). **(07 Marks)**

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