CBCS SCHEME

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17MAT31

Third Semester B.E. Degree Examination, Dec.2024/Jan.2025
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the Fourier series expansion of f(x), if $f(x) = \sqrt{1-\cos x}$, $0 < x < 2\pi$ and hence evaluate $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+.....$ **(08 Marks)**

b. Expand the function f(x) defined by, $f(x)=\begin{cases}\frac{1}{4}-x; & \text{for } 0<x<\frac{1}{2}\\ x-\frac{3}{4}; & \text{for } \frac{1}{2}<x<1\end{cases}$ in a half range sine series. **(06 Marks)**

c. A function f(x) of period 2π is specified by the following table :

x :	0	$\frac{\pi}{3}$	$2\frac{\pi}{3}$	π	$4\frac{\pi}{3}$	$5\frac{\pi}{3}$	2π
f(x) :	7.9	7.2	3.6	0.5	0.9	6.8	7.9

Obtain the Fourier series for f(x) up to the first harmonic. **(06 Marks)**

OR

2 a. Find the Fourier Series expansion of f(x). If $f(x)=\begin{cases}0 & \text{in } -\pi\le x<0\\ \sin x & 0<x\le\pi\end{cases}$. Hence deduce that $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-.......=\frac{\pi-2}{4}$ **(08 Marks)**

b. Express $f(x)=x$ as a half-range Fourier cosine series in $0<x<2$. **(06 Marks)**

c. Obtain first harmonic of the data :

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(06 Marks)

Module-2

3 a. Find the Fourier transform of $f(x)=\begin{cases}a^2-x^2 & \text{for } |x|<a\\ 0 & \text{for } |x|>a\end{cases}$, hence deduce that $\int_0^\infty \frac{\sin t - t\cos t}{t^3}dt=\frac{\pi}{4}$. **(08 Marks)**

b. Find the Fourier cosine transform of $f(x)=\frac{e^{-ax}}{x}$; $x\neq 0$, $a>0$. **(06 Marks)**

c. Solve the difference equation, using z-transform $y_{n+2}-4y_n=0$, given that $y_0=0$, $y_1=2$. **(06 Marks)**

OR

4 a. Find the Fourier transform of $f(x)=\begin{cases}1-|x|, & |x|<1\\ 0, & |x|>1\end{cases}$ and hence deduce that $\int_0^\infty \frac{\sin^2 t}{t^2}dt=\frac{\pi}{2}$. **(08 Marks)**

b. Find the inverse z-transform of $\frac{3z^2+z}{(5z-1)(5z+2)}$. **(06 Marks)**

c. Find the z-transform of, (i) $K^n \cos n\theta$ (ii) $2n+5\sin\frac{n\pi}{4}-3a^5$ **(06 Marks)**

Module-3

5 a. If θ is the angle between the two regression lines, show that $\tan\theta=\left(\frac{1-r^2}{r}\right)\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$. Explain the significance when r = 0 and r = ±1. **(08 Marks)**

b. Fit a best parabola of the form $y=a+bx+cx^2$ to the following data :

x	2	4	6	8	10
y	3.07	12.85	31.47	57.38	91.29

(06 Marks)

c. Find an approximate real root of the equation $x\sin x+\cos x=0$ near x = π, upto four decimal places using Newton's-Raphson method. **(06 Marks)**

OR

6 a. Find the co-efficient of correlation and two regression lines for the following data:

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

(08 Marks)

b. Fit a curve of the form $y=ae^{bx}$. Use the method of least squares to the following data :

x	5	15	20	30	35	40
y	10	14	25	40	50	62

(06 Marks)

c. Find the real root of the equation $\cos x=3x-1$, correct to three decimals using regula-falsi method, that lies between 6.5 and 1. (Here x is in radians) **(06 Marks)**

Module-4

7 a. From the data given in the following table, find the number of students who obtained (i) less than 45 marks (ii) between 40 and 45 marks. **(08 Marks)**

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students	31	42	51	35	31

b. Find the interpolating polynomial f(x) by using Newton's divided difference interpolation formula from the data :

x :	0	1	2	3	4	5
f(x) :	3	2	7	24	59	118

(06 Marks)

c. Compute the value of $\int_{0.2}^{1.4}(\sin x-\log_e x+e^x)dx$ using Simpson's $\frac{3}{8}^{th}$ rule by taking seven ordinates. **(06 Marks)**

OR

8 a. In the given table below, the values of y are consecutive terms of the series of which 23.6 in the 6th term. Find the first and tenth terms of the series using Newton's formulae. **(08 Marks)**

x	3	4	5	6	7	8	9
y	4.8	8.4	14.5	23.6	36.2	52.8	73.9

b. Find the interpolating polynomial f(x) by using Lagrange's formula and hence find f(3) for the following data :
$f(0) = 2$, $f(1) = 3$, $f(2) = 12$, $f(5) = 147$. **(06 Marks)**

c. Evaluate $\int_0^1 \frac{x}{1+x^2}dx$ by Weddle's rule taking seven ordinates. **(06 Marks)**

Module-5

9 a. Verify Green's theorem for $\int_C (xy + y^2)dx + x^2dy$, where C is bounded by $y = x$ and $y = x^2$. **(08 Marks)**

b. Using Stoke's theorem, evaluate $\iint_S \left(\text{curl}\,\vec{f}\right).\hat{n}dS$; $\vec{f} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - zx\hat{k}$, where S is the surface of the cube formed by the plane $x = 0$, $y = 0$, $x = 2$, $y = 2$ and $z = 2$ with its bottom removed. **(06 Marks)**

c. Solve the Euler's equation for the functional $\int_{x_0}^{x_1} (1 + x^2y')y'dx$. **(06 Marks)**

OR

10 a. State and prove Euler's equation in the form, $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(08 Marks)**

b. Use divergence theorem to evaluate $\iint_S \vec{F}.\hat{n}dS$ over the entire surface of the region above xy-plane bounded by $x = 0$, $x = 0$, $y = 0$, $y = 1$, $z = 0$, $z = 1$, where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$. **(06 Marks)**

c. Prove that the shortest distance between two points in a plane is a straight line. **(06 Marks)**

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