

Third Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025

AV Mathematics III for EC/ BM Engineering

Time: 3 hrs.

Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.

2. Statistical table and Mathematics formula handbook are allowed.

3. M : Marks, L: Bloom's level, C: Course outcomes.

Module – 1			M	L	C																		
Q.1	a.	Obtain the Fourier series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$. Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$.	6	L2	CO1																		
	b.	Find the Fourier series of $f(x) = x $ in $(-\ell, \ell)$. Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$.	7	L3	CO1																		
	c.	Expand $f(x) = 2x - 1$ as a cosine half range Fourier series in $0 < x < 1$.	7	L2	CO1																		
OR																							
Q.2	a.	Find the Fourier series of $f(x) = \begin{cases} 1 + \frac{2x}{\pi} & \text{in } -\pi < x < 0 \\ 1 - \frac{2x}{\pi} & \text{in } 0 < x < \pi \end{cases}$. Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$.	6	L2	CO1																		
	b.	Obtain the sine half range series of, $f(x) = \begin{cases} \frac{1-x}{4} & \text{in } 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \text{in } \frac{1}{2} < x < 1 \end{cases}$	7	L2	CO1																		
	c.	Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data : <table><tr><td>x°:</td><td>0</td><td>45</td><td>90</td><td>135</td><td>180</td><td>225</td><td>270</td><td>315</td></tr><tr><td>y:</td><td>2</td><td>$\frac{3}{2}$</td><td>1</td><td>$\frac{1}{2}$</td><td>0</td><td>$\frac{1}{2}$</td><td>1</td><td>$\frac{3}{2}$</td></tr></table>	x° :	0	45	90	135	180	225	270	315	y:	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	7	L1	CO1
x° :	0	45	90	135	180	225	270	315															
y:	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$															
Module – 2																							
Q.3	a.	Find the Fourier transform of the function, $f(x) = \begin{cases} 1 & \text{for } x \leq a \\ 0 & \text{for } x > a \end{cases}$. Hence evaluate $\int_0^\pi \frac{\sin x}{x} dx$.	6	L2	CO2																		
	b.	Find the Fourier sine and cosine transforms of $f(x) = e^{-\alpha x}$, $\alpha > 0$.	7	L2	CO2																		
	c.	Find the Fourier sine transform of $\frac{e^{-ax}}{x}$, $a > 0$.	7	L3	CO2																		

OR																		
Q.4	a.	If $f(x) = \begin{cases} 1-x^2, & x < 1 \\ 0, & x \geq 1 \end{cases}$, find the Fourier transform of $f(x)$ and hence find the value of, $\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} dx$.	6	L2	CO2													
	b.	Find the Fourier sine transform of $f(x) = e^{- x }$ and hence evaluate $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx$, $m > 0$.	7	L3	CO2													
	c.	Find the Discrete fast fourier of signal $= (0, 1, 49)^T$	7	L3	CO2													
Module – 3																		
Q.5	a.	Find the z-transform of, (i) $\cosh n\theta$ (ii) $\sinh n\theta$	6	L1	CO3													
	b.	If $V(z) = \frac{2z^2 + 3z + 12}{(z-1)^4}$, evaluate u_0, u_1 and u_2	7	L2	CO3													
	c.	Find the inverse z-transform of, $\frac{z}{(z-1)(z-2)}$.	7	L2	CO3													
OR																		
Q.6	a.	Solve by using z-transforms, $y_{n+2} + 2y_{n+1} + y_n = n$ with $y_0 = 0 = y_1$	6	L3	CO3													
	b.	Find $z^{-1} \left[\frac{5z}{(3z-1)(2-z)} \right]$.	7	L2	CO3													
	c.	Solve by using z-transforms $u_{n+2} - 5u_{n+1} + 6u_n = 2^n$ with $u_0 = 0 = u_1$.	7	L3	CO3													
Module – 4																		
Q.7	a.	Solve $\frac{d^3 y}{dx^3} + 6\frac{d^2 y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$.	6	L1	CO4													
	b.	Solve $(D^2 + 1)y = x^2 + 4x - 6$.	7	L2	CO4													
	c.	Using the method of variation of Parameters of $\frac{d^2 y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}$	7	L3	CO4													
OR																		
Q.8	a.	Solve $6\frac{d^2 y}{dx^2} + 17\frac{dy}{dx} + 12y = e^{-x}$.	6	L2	CO4													
	b.	Solve the Cauchy's differential equation, $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + 8y = 65 \cos(\log x)$.	7	L2	CO4													
	c.	The charge q in a series circuit containing an Inductance L , Capacitance C , emf E satisfy the differential equation, $L \frac{d^2 q}{dt^2} + \frac{q}{C} = E$. Express q in terms of t .	7	L3	CO4													
Module – 5																		
Q.9	a.	Fit a second degree parabola $y = a + bx + cx^2$ into least square sense for the data and estimate y at $x = 6$. <table><tr><td>x:</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y:</td><td>10</td><td>12</td><td>13</td><td>16</td><td>19</td></tr></table>	x:	1	2	3	4	5	y:	10	12	13	16	19	6	L1	CO5	
x:	1	2	3	4	5													
y:	10	12	13	16	19													

	b.	Find a correlation coefficient for the two variables x and y.	7	L2	CO5																						
		<table><tr><td>x:</td><td>92</td><td>89</td><td>87</td><td>86</td><td>83</td><td>77</td><td>71</td><td>63</td><td>53</td><td>50</td></tr><tr><td>y:</td><td>86</td><td>83</td><td>91</td><td>77</td><td>68</td><td>85</td><td>52</td><td>82</td><td>37</td><td>57</td></tr></table>	x:	92	89	87	86	83	77	71	63	53	50	y:	86	83	91	77	68	85	52	82	37	57			
x:	92	89	87	86	83	77	71	63	53	50																	
y:	86	83	91	77	68	85	52	82	37	57																	
	c.	Ten students got the following percentage of marks in two subjects x and y. Compute the rank correlation coefficient.	7	L2	CO5																						
		<table><tr><td>x:</td><td>78</td><td>36</td><td>98</td><td>25</td><td>75</td><td>82</td><td>90</td><td>62</td><td>65</td><td>39</td></tr><tr><td>y:</td><td>84</td><td>51</td><td>91</td><td>60</td><td>68</td><td>62</td><td>86</td><td>58</td><td>53</td><td>47</td></tr></table>	x:	78	36	98	25	75	82	90	62	65	39	y:	84	51	91	60	68	62	86	58	53	47			
x:	78	36	98	25	75	82	90	62	65	39																	
y:	84	51	91	60	68	62	86	58	53	47																	
OR																											
Q.10	a.	If θ is the angle between the lines of regression show that $\tan \theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \left(\frac{1 - r^2}{r} \right).$	6	L2	CO5																						
	b.	Obtain the lines of regression and hence find the coefficient of correlation for the data, <table><tr><td>x:</td><td>1</td><td>3</td><td>4</td><td>2</td><td>5</td><td>8</td><td>9</td><td>10</td><td>13</td><td>15</td></tr><tr><td>y:</td><td>8</td><td>6</td><td>10</td><td>8</td><td>12</td><td>16</td><td>16</td><td>10</td><td>32</td><td>32</td></tr></table>	x:	1	3	4	2	5	8	9	10	13	15	y:	8	6	10	8	12	16	16	10	32	32	7	L2	CO5
x:	1	3	4	2	5	8	9	10	13	15																	
y:	8	6	10	8	12	16	16	10	32	32																	
	c.	If $8x - 10y + 66 = 0$ and $40x - 18y = 214$ are the two regression lines. Find $\bar{x}$, $\bar{y}$ and r. Find σ_y if $\sigma_x = 3$.	7	L2	CO5																						

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