CBCS SCHEME

USN | | | | | | | | | | |

21MAT31

Third Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the Laplace transform of $te^{-2t}\sinh 3t$ **(06 Marks)**

b. Find the Laplace transform of the triangular wave of period '2a' given by

$$f(t)=\begin{cases} t & \text{in } 0<t<a \\ 2a-t & \text{in } a<t<2a \end{cases}$$ **(07 Marks)**

c. Using convolution theorem, find the inverse Laplace transform of $\dfrac{1}{s(s^2+a^2)}$ **(07 Marks)**

OR

2 a. Using unit step function, find the Laplace transform of

$$f(t)=\begin{cases} 1 & \text{in } 0<t<1 \\ t & \text{in } 1<t\le 2 \\ t^2 & \text{in } t>2 \end{cases}$$ **(06 Marks)**

b. Find the inverse Laplace transform of

(i) $\dfrac{4s-18}{9-s^2}$ (ii) $\dfrac{s-2}{s^2+7s+12}$ **(07 Marks)**

c. Solve by using Laplace transform techniques ,

$$\frac{d^2y}{dx^2}+5\frac{dy}{dx}+6y=5e^{2x}, \quad \text{given } y(0)=2,\ y'(0)=1.$$ **(07 Marks)**

Module-2

3 a. Obtain Fourier series expansion of $f(x)=x(2\pi-x)$ in $0\le x\le 2\pi$ **(06 Marks)**

b. Obtain the half range cosine series for the function $f(x)=x^2$, over the interval in $(0,\pi)$. **(07 Marks)**

c. Obtain the constant term and coefficient of first cosine and sine terms in the Fourier series expansion of y from the table.

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(07 Marks)

OR

4 a. Find the Fourier series expansion of $f(x) = 2x - x^2$ in $(0, 2)$ **(06 Marks)**

b. Find sine half range series of f(x) in (0, 1) where $f(x) = \begin{cases} \frac{1}{4} - x & \text{in } (0, 1/2) \\ x - \frac{3}{4} & \text{in } (1/2, 1) \end{cases}$ **(07 Marks)**

c. Obtain Fourier series expansion upto first harmonics of y = f(x). Also find their amplitudes for the following data :

x	0	60	120	180	240	300	360
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(07 Marks)

Module-3

5 a. Find the Fourier transform of

$$f(x) = \begin{cases} 1 & \text{in } |x| \le a \\ 0 & \text{in } |x| > a \end{cases}$$ **(06 Marks)**

b. Find the Fourier cosine and sine transform of $f(x) = 2e^{-3x}$. **(07 Marks)**

c. Find the Z-transform of $\cos n\theta$ and $\sin n\theta$. **(07 Marks)**

OR

6 a. Find the complex fourier transform of the function

$$f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$$

Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$ **(06 Marks)**

b. Find the inverse Z – transform of $\dfrac{3z^2 - 8z}{(z-2)(z-3)}$ **(07 Marks)**

c. Using Z – transform, solve the difference equation $y_{n+2} - 5y_{n+1} + 6y_n = u_n$ when $y_0 = 0$, $y_1 = 1$ and $u_n = 1$. **(07 Marks)**

Module-4

7 a. Classify the following partial differential equation

(i) $u_{xx} + u_{xy} + (x^2 + 4y^2)u_{yy} = 2 \sin xy$

(ii) $x^2u_{yy} + 2xy\, u_{xy} + y^2 u_{xx} = 0$

(iii) $(1 + x^2)u_{xx} + (5 + 2x^2)u_{xt} + (4 + x^2)u_{tt} = 0$

(iv) $x^2u_{xx} + (1 - y^2)u_{yy} = 0 \quad -\infty < x < \infty; \ -1 < y < 1$ **(10 Marks)**

b. Solve the wave equation $\dfrac{\partial^2 u}{\partial t^2} = 4\dfrac{\partial^2 u}{\partial x^2}$ subject to the condition $u(0, t) = 0$, $u(4, t) = 0$, $u_t(x, 0) = 0$ and $u(x, 0) = x(4 - x)$ by taking $h = 1, k = 0.5$ upto four steps. **(10 Marks)**

OR

8 a. Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$, taking $h = 1$ upto $t = 1.25$. The boundary conditions are $u(0, t) = u(5, t) = 0$, $u_t(x, 0) = 0$ and $u(x, 0) = x^2(5 - x)$. **(10 Marks)**

b. Given the values of $u(x, y)$ on the boundary of the square in the figure. Evaluate the function $u(x, y)$ satisfying the Laplace equation $u_{xx} + u_{yy} = 0$ at the pivotal points of the Fig.Q8(b).

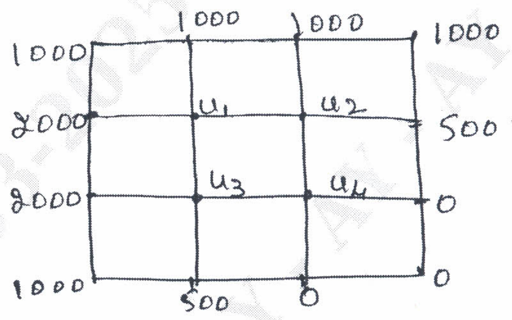

Fig.Q8(b) **(10 Marks)**

Module-5

9 a. Using Runge-Kutta method of order four, solve

$$\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2 \quad \text{for } x = 0.2. \text{ Given that } y(0) = 1,\ y'(0) = 0$$ **(06 Marks)**

b. Derive Euler's equation. **(07 Marks)**

c. Prove that the geodesics on a plane are straight line. **(07 Marks)**

OR

10 a. Apply Milne's method to compute $y(0.8)$ given that

$$\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx} \quad \text{and the initial values are}$$

$y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$

$y'(0) = 0$, $y'(0.2) = 0.1996$, $y'(0.4) = 0.3937$, $y'(0.6) = 0.5689$. **(06 Marks)**

b. Find the extremal of the functional $\int_{x_1}^{x_2} (y' + x^2 y')^2\, dx$ **(07 Marks)**

c. Solve the variational problem.

$$\delta\int_0^1 (x + y + y'^2)\, dx = 0 \quad \text{under the conditions } y(0) = 1;\ y(1) = 2.$$ **(07 Marks)**

* * * * *