



CBCS SCHEME

18MAT21

Second Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025 Advanced Calculus and Numerical Methods

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ along $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$. (06 Marks)
- b. If $\vec{F} = \nabla(xy^3z^2)$ find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ at $(1, -1, 1)$. (07 Marks)
- c. Find the value of the constant 'a' such that the vector field, $\vec{F} = (axy - z^3)\mathbf{i} + (a - 2)x^2\mathbf{j} + (1 - a)xz^2\mathbf{k}$ is irrotational and hence find a scalar function ϕ such that $\vec{F} = \nabla\phi$. (07 Marks)

OR

- 2 a. If $\vec{F} = (3x^2 + 6y)\mathbf{i} - 14yz\mathbf{j} + 20xz^2\mathbf{k}$, evaluate $\int \vec{F} \cdot d\vec{r}$ from $(0, 0, 0)$ to $(1, 1, 1)$ along the curve given by $x = t, y = t^2, z = t^3$. (06 Marks)
- b. Find the area between the parabola $y^2 = 4x$ and $x^2 = 4y$ using Green's theorem. (07 Marks)
- c. Evaluate $\int_C xy \, dx + xy^2 \, dy$ by using Stoke's theorem where C is the square in the xy -plane with vertices $(1, 0), (-1, 0), (0, 1)$ and $(0, -1)$. (07 Marks)

Module-2

- 3 a. Solve $(D^3 + 6D^2 + 11D + 6)y = 0$ (06 Marks)
- b. Solve $\frac{d^2y}{dx^2} + y = \sec x \tan x$ by the method of variation of parameters. (07 Marks)
- c. Solve $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = (1 + x)^2$ (06 Marks)

OR

- 4 a. Solve $(D^2 + 1)y = e^x + x^4 + \sin x$ (06 Marks)
- b. Solve $(1 + x)^2 \frac{d^2y}{dx^2} + (1 + x) \frac{dy}{dx} + y = \sin 2\{\log(1 + x)\}$ (07 Marks)
- c. The current i and the charge q in a series containing an inductance L , capacitance C , emf ϵ , satisfy the differential equation $L \frac{d^2q}{dt^2} + \frac{q}{C} = \epsilon$. Find q and i terms of 't' given that L, C, ϵ are constants and the value of i and q are zero initially. (07 Marks)

Module-3

- 5 a. Form the partial differential equation by elimination of arbitrary function from $\phi(x + y + z, xy + z^2) = 0$ (06 Marks)
- b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2 \sin y$, when $x = 0$ and $z = 0$, when y is odd multiple of $\pi/2$. (07 Marks)
- c. Derive one dimensional heat equation in the form $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ (07 Marks)

OR

- 6 a. Solve $\frac{\partial^2 z}{\partial y^2} = z$ given that $z = 0$ and $\frac{\partial z}{\partial y} = \sin x$ when $y = 0$. (06 Marks)
- b. Solve $(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} + (mx - ly) = 0$ (07 Marks)
- c. Find all possible solutions of one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ by variable separable method. (07 Marks)

Module-4

- 7 a. Test for convergence for series $1 + \frac{2}{3}x + \frac{2.3}{3.5}x^2 + \frac{2.3.4}{3.5.7}x^3 + \dots (x > 0)$ (06 Marks)
- b. Solve Bessel's differential equation $x^2 y'' + x y' + (x^2 - n^2)y = 0$ leading to $J_n(x)$. (07 Marks)
- c. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. (07 Marks)

OR

- 8 a. Discuss the nature of the series $\sum \frac{(n+1)^n x^n}{n^{n+1}}$ (06 Marks)
- b. If α and β are two distinct roots of $J_n(x) = 0$, prove that $\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = 0$, if $\alpha \neq \beta$. (07 Marks)
- c. If $x^3 + 2x^2 - x + 1 = aP_0(x) + bP_1(x) + cP_2(x) + dP_3(x)$, find the values of a , b and c . (07 Marks)

Module-5

- 9 a. Using Newton-Raphson method find the real root of the equation $3x = \cos x + 1$. Carry out three iterations. (06 Marks)
- b. Using Newton's forward interpolation formula find $f(3)$ given :

x	0	2	4	6	8	10
$y = f(x)$	0	4	56	204	496	980

(07 Marks)

- c. Evaluate $\int_0^1 \frac{dx}{1+x}$ by applying Simpson's $3/8^{\text{th}}$ rule considering 6 equal subintervals. Hence deduce the value of $\log_e 2$ (07 Marks)

OR

- 10 a. Using Newton's divided difference formula find $f(8)$ from the following data:

x :	4	5	7	10	11	13
y :	48	100	294	900	1210	2028

(06 Marks)

- b. Using Regula-Falsi method find the real root of $x \log_{10} x - 1.2 = 0$. Carry out three iterations. (07 Marks)

- c. Evaluate $\int_4^{5.2} \log_e x \, dx$ taking 6 equal strips by applying Weddle's rule. (07 Marks)

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