

Second Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025 Mathematics – II for CSE Stream

Time: 3 hrs.

Max. Marks: 100

- Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.*

Module – 1				M	L	C
Q.1	a.	Evaluate $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx$	07	L2	CO1	
	b.	Prove that $\beta(m,n) = \frac{\overline{m} \cdot \overline{n}}{(m+n)}$	07	L2	CO1	
	c.	Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dy dx$ by changing to polar coordinates.	06	L3	CO1	
OR						
Q.2	a.	Evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy dy dx$ by change the order of integration.	07	L2	CO1	
	b.	Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} \times \int_0^{\pi/2} \sqrt{\sin \theta} d\theta = \pi$	07	L2	CO1	
	c.	Write a program to find the volume of the tetrahedron bounded by the planes $x=0, y=0, z=0, \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.	06	L3	CO5	
Module – 2						
Q.3	a.	Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at the point $(2, -1, 2)$ along $2\hat{i} - 3\hat{j} + 6\hat{k}$.	07	L2	CO2	
	b.	Find the value of the constants a, b, c such that $\vec{F} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$ is irrotational.	07	L2	CO2	
	c.	Show that the cylindrical co-ordinate system is orthogonal.	06	L3	CO2	
OR						
Q.4	a.	Find the value of the constants 'a' such that the vector field, $\vec{F} = (axy - z^3)\hat{i} + (a-2)x^2\hat{j} + (1-a)xz^2\hat{k}$ is irrotational.	07	L2	CO2	
	b.	Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.	07	L2	CO2	
	c.	Write a program to verify whether the following vectors $(2, 1, 5, 4)$ and $(3, 4, 7, 8)$ are orthogonal.	06	L3	CO5	

Module – 3

Q.5	a. Express the matrix $A = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$ in the vector spaces of 2×2 matrices as a linear combination of $B = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$	07	L2	CO3
	b. Determine whether the vectors $V_1 = (1, 2, 3)$, $V_2 = (3, 1, 7)$ and $V_3 = (2, 5, 8)$ are linearly dependent or linearly independent.	07	L2	CO3
	c. Verify the rank nullity theorem for the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$	06	L3	CO3

OR

Q.6	a. Let W be the subspace of \mathbb{R}^5 spanned by $x_1 = (1, 2, -1, 3, 4)$, $x_2 = (2, 4, -2, 6, 8)$, $x_3 = (1, 3, 2, 2, 6)$, $x_4 = (1, 4, 5, 1, 8)$ and $x_5 = (2, 7, 3, 3, 9)$. Find a subset of vectors which forms a basis of W .	07	L2	CO3
	b. Consider the following polynomials in $p(t)$ and inner product : $f(t) = t + 2$, $g(t) = 3t - 2$, $h(t) = t^3 - 2t - 3$ and $\langle f, g \rangle = \int_0^1 f(t)g(t) dt$. (i) Find $\langle f, g \rangle$ and $\langle f, h \rangle$ (ii) Find $\ f\ $ and $\ g\ $	07	L2	CO3
	c. If V is a vector space of polynomials over \mathbb{R} . Find a basis and dimension of the subspaces W and V , spanned by the polynomials. $x_1 = t^3 - 2t^2 + 4t + 1$, $x_2 = 2t^3 - 3t^2 + 9t - 1$ $x_3 = t^3 + 6t - 5$, $x_4 = 2t^3 - 5t^2 + 7t + 5$	06	L2	CO3

Module – 4

Q.7	a.	Find the real root of the equation $x \log_{10} x - 1.2 = 0$ by Regular Falsi method. Correct to four decimal places.	07	L2	CO4												
	b.	From the following table find the number of students who have obtained less than 45 marks. <table><tr><td>Marks</td><td>30 – 40</td><td>40 – 50</td><td>50 – 60</td><td>60 – 70</td><td>70 – 80</td></tr><tr><td>No. of students</td><td>31</td><td>42</td><td>51</td><td>35</td><td>31</td></tr></table>	Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	No. of students	31	42	51	35	31	07	L2	CO4
Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80												
No. of students	31	42	51	35	31												
	c.	Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $(1/3)^{\text{rd}}$ rule taking four equal strips.	06	L3	CO4												

OR

Q.8	a.	Fit the polynomial for the following data using Newton's divided difference formula and hence find $f(3)$.	07	L2	CO4														
		<table><tr><td>x</td><td>2</td><td>4</td><td>5</td><td>6</td><td>8</td><td>10</td></tr><tr><td>y</td><td>10</td><td>96</td><td>196</td><td>350</td><td>868</td><td>1746</td></tr></table>				x	2	4	5	6	8	10	y	10	96	196	350	868	1746
		x				2	4	5	6	8	10								
y	10	96	196	350	868	1746													
	b.	Using Lagrange's interpolation formula find $f(4)$.	07	L2	CO4														
		<table><tr><td>x</td><td>0</td><td>2</td><td>3</td><td>6</td></tr><tr><td>y</td><td>- 4</td><td>2</td><td>14</td><td>158</td></tr></table>	x	0	2	3	6	y	- 4	2	14	158							
x	0	2	3	6															
y	- 4	2	14	158															

Q.8	c.	Use Simpson's $(3/8)^{\text{th}}$ rule to evaluate $\int_1^4 e^{1/x} dx$ by taking four ordinates.	06	L3	CO4
Module – 5					
Q.9	a.	Employ Taylor's series method to solve the initial value problem $\frac{dy}{dx} = x - y^2$; $y(0) = 1$ at the point $x = 0.1$ by considering upto 4^{th} degree terms.	07	L2	CO4
	b.	Apply Milne's method to compute $y(1.4)$ for the differential equation $\frac{dy}{dx} = x^2 + \frac{y}{2}$, given that $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.3) = 2.4649$ and $y(1.3) = 2.7514$ correct to four decimal places.	07	L2	CO4
	c.	Use fourth order Runge Kutta method to find the value of y at $x = 0.1$, given that $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 0$ and $h = 0.1$.	06	L2	CO4
OR					
Q.10	a.	Use Modified Euler's method to compute $y(0.1)$, given that $\frac{dy}{dx} = x^2 + y$; $y(0) = 1$ by taking $h = 0.05$.	07	L2	CO4
	b.	If $\frac{dy}{dx} = 2e^x - y$; $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$ and $y(0.3) = 2.090$. Find the value of y at $x = 0.4$ correct to four decimal places by applying Milne's predictor and corrector method.	07	L2	CO4
	c.	Write a program to solve : $\frac{dy}{dx} - 2y = 3e^x$ with $y(0) = 0$ using Taylor's series method at $x_1 = 0.1$, $x_2 = 0.2$ and $x_3 = 0.3$.	06	L3	CO5
