CBCS SCHEME

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18MAT11

First Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025

Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. With usual notation, prove that $\tan\phi = r\dfrac{d\theta}{dr}$. **(06 Marks)**

b. Find the radius of curvature for the Folium of De-Cartes $x^3 + y^3 = 3axy$ at the point $\left(\dfrac{3a}{2}, \dfrac{3a}{2}\right)$ on it. **(06 Marks)**

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$. **(08 Marks)**

OR

2 a. Show that the pair of curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$ intersect each other orthogonally. **(06 Marks)**

b. Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$ **(06 Marks)**

c. Show that for the curve $r = a(1 + \cos\theta)$, $\dfrac{\rho^2}{r}$ is a constant. **(08 Marks)**

Module-2

3 a. Using Maclaurin's series prove that

$$\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} \ldots\ldots$$
(06 Marks)

b. Evaluate (i) $\lim_{x\to 1} x^{1/1-x}$ (ii) $\lim_{x\to\pi/2} (\cos x)^{\pi/2 - x}$ **(07 Marks)**

c. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ **(07 Marks)**

OR

4 a. If $u = f(x - y, y - z, z - x)$ show that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$. **(06 Marks)**

b. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$ find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1, -1, 0)$. **(07 Marks)**

c. Find the maximum and minimum distance of the point $(1, 2, 3)$ from the sphere $x^2 + y^2 + z^2 = 56$. **(07 Marks)**

Module-3

5 a. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dz\,dy\,dx$ **(06 Marks)**

b. Find by double integration the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. **(07 Marks)**

c. Show that $\beta(m,n) = \dfrac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$ **(07 Marks)**

OR

6 a. Evaluate $\int_{x=0}^{\infty}\int_{y=x}^{\infty}\dfrac{e^{-y}}{y}\,dy\,dx$ by changing the order of integration. **(06 Marks)**

b. Find the volume generated by the revolution of the cardioide $r = a(1 + \cos\theta)$ about the initial line. **(07 Marks)**

c. Show that $\int_{0}^{\pi/2}\dfrac{d\theta}{\sqrt{\sin\theta}}\int_{0}^{\pi/2}\sqrt{\sin\theta}\cdot d\theta = \pi$ **(07 Marks)**

Module-4

7 a. Solve $(x^2 + y^2 + x)\,dx + xy\,dy = 0$ **(06 Marks)**

b. Find the orthogonal trajectories of the family of curves $r^n = a^n \cos n\theta$. **(07 Marks)**

c. If the temperature of the air is 30°C and a metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach a temperature of 40°C. **(07 Marks)**

OR

8 a. Solve $y(2xy + e^x)\,dx - e^x\,dy = 0$ **(06 Marks)**

b. Solve the equation $y^2(y - xp) = x^4p^2$ by reducing into Clairaut's form, taking the substitution $X = \dfrac{1}{x}$ and $Y = \dfrac{1}{y}$. **(07 Marks)**

c. A series circuit with resistance R, inductance L and electromotive force E is governed by the differential equation $L\dfrac{di}{dt} + Ri = E$, where L and R are constant and initially the current i is zero. Find the current at any time t. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

by applying elementary row operations. **(06 Marks)**

b. Apply Gauss-Jordon method to solve the following system of equations:

$$2x + y + 3z = 1$$
$$4x + 4y + 7z = 1$$
$$2x + 5y + 9z = 3$$

(07 Marks)

c. Find the dominant eigen value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

by powers method taking the initial eigen vector as $[1, 1, 1]^1$. Carry out 5 iterations. **(07 Marks)**

OR

10 a. Investigate the values of λ and μ such that the system of equations

$$x + y + z = 6, \quad x + 2y + 3z = 10, \quad x + 2y + \lambda z = \mu$$

may have (i) unique solution (ii) infinite solution (iii) No solution. **(06 Marks)**

b. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form. **(07 Marks)**

c. Solve the following system of equations by Gauss Seidel method. Carry out three iterations.

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

(07 Marks)

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