Hamiltonian Laceability in Line Graphs

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ABSTRACT
A Connected graph G is a Hamiltonian laceable if there exists in G a Hamiltonian path between every pair of vertices in G at an odd distance. G is a Hamiltonian-t-Laceable (Hamiltonian-t*-Laceable) if there exists in G a Hamiltonian path between every pair (at least one pair) of vertices at distance'‘t’ in G. 1≤ t ≤ diamG. In this paper we explore the Hamiltonian-t*-laceability number $(a*_{(t)})$ of graph L (G) i.e., Line Graph of G and also explore Hamiltonian-t*-Laceable of Line Graphs of Sunlet graph, Helm graph and Gear graph for t=1,2 and 3.

Keywords
Connected graph, Line graph, Sun let graph, Helm graph, Wheel graph, Gear graph and Hamiltonian-t-laceable graph.

1. INTRODUCTION
All graphs considered here are finite, simple, connected and undirected graph. Let $(G = V(G), E(G))$ be a graph. $|V(G)|$ and $|E(G)|$ are called the order and the size of G respectively. The order of G denoted by O(G) is the cardinality of vertices of G. The distance between u and v denoted by d(u,v) is the length of the shortest u-v path in G. G is a Hamiltonian path between every pair of the distinct vertices in it at an odd distance. G is a Hamiltonian-t-laceable if there exists a Hamiltonian path between every pair of the vertices u and v in G with the property d(u,v)=t, where t is a positive integer, such that 1≤ t ≤ diamG.

The Line graph L(G) of G has the edges of G as its vertices and two vertices of L(G) are adjacent if and only if they are adjacent in G. In [3],[5],[6] and [7] the authors have studied Hamiltonian-t-laceability and Hamiltonian-t*laceability of various graph structures. In this paper we explore the Hamiltonian-t*-laceability number of Line graph L(G) and also Hamiltonian-t*-laceability of Line graph L(G) of the sun let graph, Helm graph and Gear graph.

DEFINITION 1
The Line graph L(G) of G is the graph of E in which $x, y \in E$ are adjacent as vertices if and only if they are adjacent as edges in G. In Figure 1, we display the graph G and its Line graph L (G).

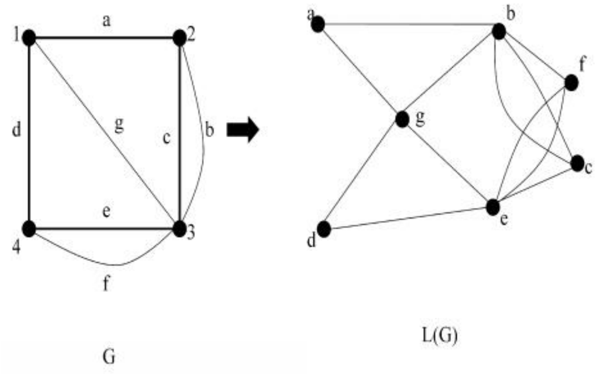

Figure 1

DEFINITION 2
The Sun let graph S_n is a graph with cycle where by each vertex of the cycle is attached to one pendent vertex. Each sun let graph contains r-vertices with r-edges.

In Figure 2, we display the Sun let graph S_n

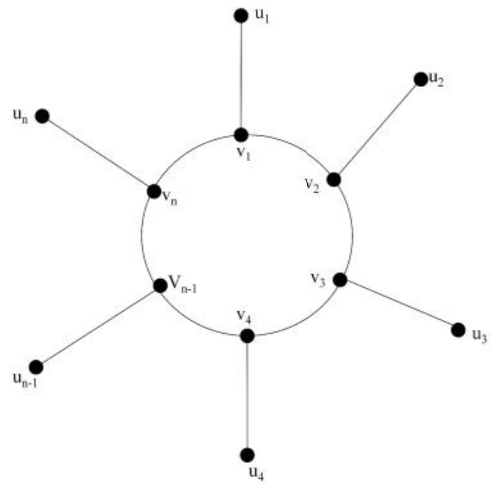

Figure 2

DEFINITION 3
The wheel graph with n spokes, W_n is the graph that consists of an n-cycle and one additional vertex, say u, which is adjacent to all the vertices of the cycle.
In Figure 3, we display the Wheel graph W_6.

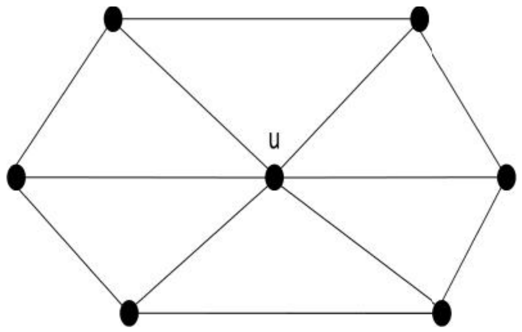

Figure 3

DEFINITION 4

The Helm graph H_n is a graph obtained from an n-wheel graph by adjoining a pendent edge at each node of the cycle. In Figure 4, we display the Helm graph H_n.

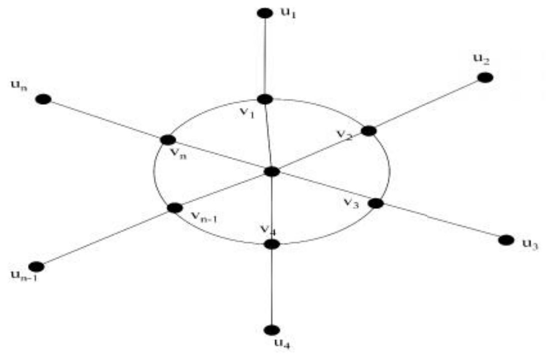

Figure 4

DEFINITION 5

The Gear graph G_n is a wheel graph with a vertex added between each pair adjacent graph vertices of the outer cycle. The Gear graph G_n has 2n+1 vertices and 3n edges. In Figure 5, we display the Gear graph G_n.

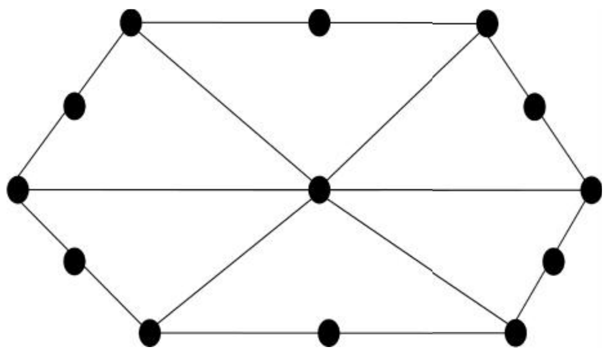

Figure 5

DEFINITION 6

For a connected graph G, the t-laceability number $a_{(t)}$ (t*laceabilty number $a*_{(t)}$) is defined as the minimum number of edges to be added to G such that there exist a Hamiltonian path between every pair (at least one pair) of vertices u and v in G with the property d(u, v) = t where t is positive integer.

2. RESULTS

Theorem 2.1: The Line graph L (G), where G=S_n, the sun let graph is Hamiltonian-t*-laceable for t=1 and 2 if odd n ≥ 3, where 1≤ t ≤ diamG.

Proof: Consider the graph G=S_n, the Line graph L(S_n) denote the vertices L(G) by

$a_1, b_1, a_2, b_2, a_3, b_3, ---a_{n-1}, b_{n-1}, a_n, b_n$ for t= 1, 2

Case (i): For t=1

In L(S_n), we find that $d(a_1, b_1) = 1.$ and the path

$$P:(a_1,b_n) \cup (b_n,a_n) \cup (a_n,b_{n-1}) \cup (b_{n-1},a_{n-1}) \cup (a_{n-1},b_{n-2}) \cup (a_{n-2},b_{n-3}) \cup ------- \cup (b_{n-6},a_{n-6}) \cup --------- \cup (b_3,a_3) \cup (a_3,b_2) \cup (b_2,a_2) \cup (a_2,b_1)$$

is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1, b_1) = 1.$ Therefore G is a Hamiltonian-t*-laceable for t=1.

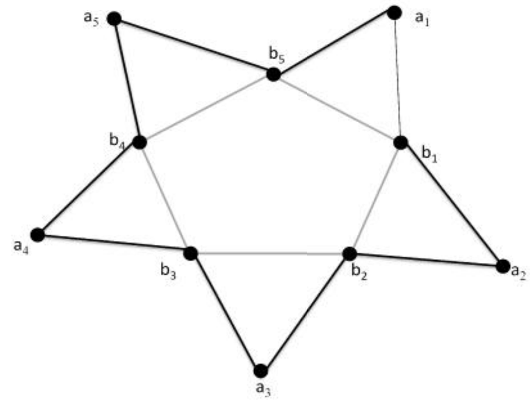

Figure 6: Hamiltonian path from the vertex a_1 to b_1 in Line graph L[S_n]

Case (ii): For t=2

In L(S_n), we find that $d(a_1, a_2) = 2.$ and the path

$$P:(a_1,b_n) \cup (b_n,a_n) \cup (a_n,b_{n-1}) \cup (b_{n-1},a_{n-1}) \cup (a_{n-1},b_{n-2}) \cup (a_{n-2},b_{n-3}) \cup ------- \cup (b_{n-6},a_{n-6}) \cup --------- \cup (b_3,a_3) \cup (a_3,b_2) \cup (b_2,b_1) \cup (b_1,a_2)$$

is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1, a_2) = 2.$. Therefore G is a Hamiltonian-t*-laceable for t=2.

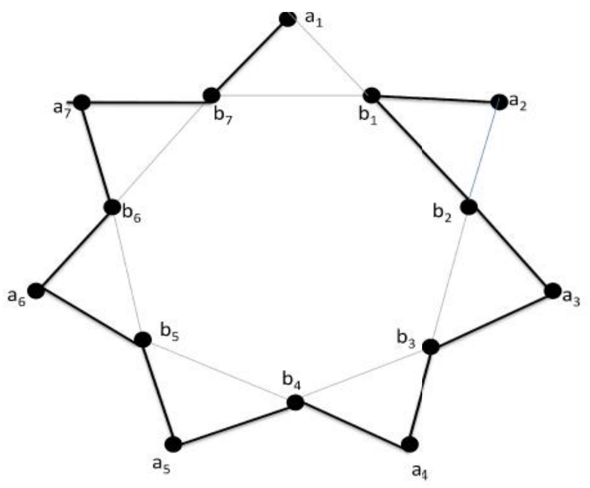

Figure 7: Hamiltonian path from the vertex a_1 to a_2 in Line graph $L[S_7]$

Lemma 2.1.1: The Line graph L(G), where $G=S_n$, is a Hamiltonian-t-laceability number if $(\text{ª} * (t))$ =1 for t=2 if odd $n \geq 3$ and t=3 if odd $n \geq 5$ where $1 \leq t \leq diamG$.*

Proof: Consider the graph $G=S_n$, its line $L(S_n)$. Here we need to establish the following cases to show that, Hamiltonian-t*-laceability number if $(\text{ª} * (t))$ =1 for t=2 if $n \geq 3$ and t=2 and 3 if $n \geq 5$

Case (i): For t=2

In $L(S_n)$, we find that $d(a_1, b_2) = 2$ and the path

$$P:(a_1,b_1)\cup(b_1,b_n)\cup(b_n,a_n)\cup(a_n,b_{n-1})\cup(b_{n-1},a_{n-1})\cup(a_{n-1},b_{n-2})\cup(b_{n-2},a_{n-2})\cup---\cup(a_{n-9},b_{n-10})\cup(b_{n-10},a_{n-10})\cup(a_{n-10},n_{n-11})\cup(b_{n-11},a_{n-11})\cup(a_{n-11},b_{n-12})\cup---------\cup(b_3,a_3)\cup(a_3,a_2)\cup(a_2,b_2)$$

is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1, b_2) = 2$ Therefore G is a Hamiltonian-t*-laceable for t=2 and Laceability number $(\text{ª} * (t))$ =1 for t=2.

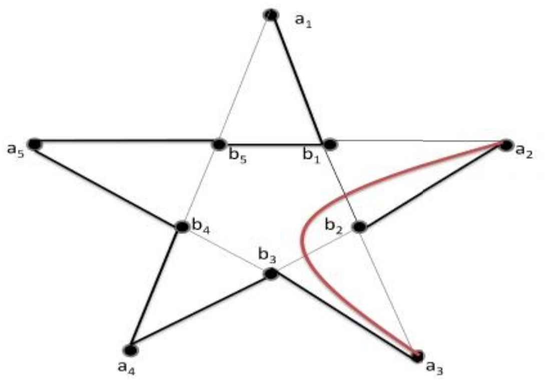

Figure 8: Hamiltonian path from the vertex a_1 to b_2 in Line graph $L[S_5]$

Case (ii): For t=3 if odd $n \geq 5$

In $L(S_n)$, we find that $d(a_1,b_3)=3$ and the path

$$P:(a_1,b_1)\cup(b_1,a_2)\cup(a_2,b_2)\cup(b_2,a_3)\cup(a_3,a_n)\cup(a_n,b_n)\cup---\cup(a_{n-2},b_{n-3})\cup(b_{n-3},a_{n-3})\cup------\cup(b_{n-10},a_{n-10})\cup(a_{n-10},b_{n-11})\cup(b_{n-11},a_{n-11})\cup(a_{n-11},b_{n-12})\cup(a_{n-11},b_{n-12})\cup--------\cup(b_5,b_4)\cup(b_4,a_4)\cup(a_4,a_3)$$

is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1,b_3)=3$. Therefore G is a Hamiltonian-t*-laceable for t=3 and Laceability number $(\text{ª} * (t))$ =1 for t=3.

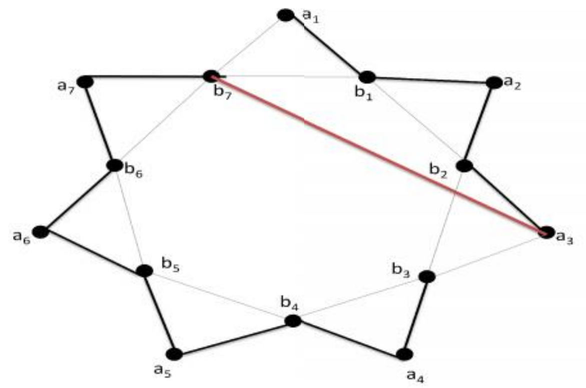

Figure 9: Hamiltonian path from the vertex a_1 to b_3 in Line graph $L[S_7]$

Theorem 2.2: The Line graph L (G), where $G=S_n$, the sun let graph is Hamiltonian-t*-laceable for t=1,2 and 3 if even $n \geq 4$, where $1 \leq t \leq diamG$.

Proof: Consider the graph $G=S_n$, the Line graph $L(S_n)$ denote the vertices L(G) by

$a_1, b_1, a_2, b_2, a_3, b_3, ---a_{n-1}, b_{n-1}, a_n, b_n$ for t= 1,2 and 3

Case (i): For t=1

In $L(S_n)$, we find that $d(a_1,b_1)=1$ and the path

$$P:(a_1,b_n)\cup(b_n,a_n)\cup(a_n,b_{n-1})\cup(b_{n-1},a_{n-1})\cup(a_{n-1},b_{n-2})\cup(b_{n-2},a_{n-2})\cup(a_{n-2},b_{n-3})\cup(b_{n-3},a_{n-3})\cup------\cup(b_{n-6},a_{n-6})\cup----\ -------\cup(b_3,a_3)\cup(a_3,b_2)\cup(b_2,a_2)\cup(a_2,b_1)$$

is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1, b_1) = 1$. Therefore G is a Hamiltonian-t*-laceable for t=1.

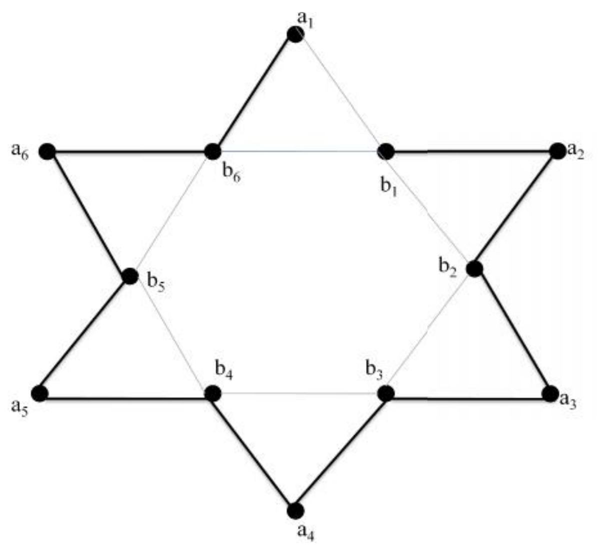

Figure 10: Hamiltonian path from the vertex a_1 to b_1 in Line graph $L[S_6]$

Case (ii): For t=2

In $L(S_n)$, we find that $d(a_1,a_2)=2$ and the path

$$P:(a_1,b_1)\cup(b_1,b_n)\cup(b_n,a_n)\cup(a_n,b_{n-1})\cup(b_{n-1},a_{n-1})\cup(a_{n-1},b_{n-2})\cup---------\cup(a_{n-5},b_{n-6})\cup------\cup(b_{n-14},a_{n-14})\cup----\ --------\cup(b_4,a_4)\cup(a_4,b_3)\cup(b_3,a_3)\cup(a_3,b_2)\cup(b_2,a_2)$$

is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1,a_2)=2$. Therefore G is a Hamiltonian-t*-laceable for t=2.

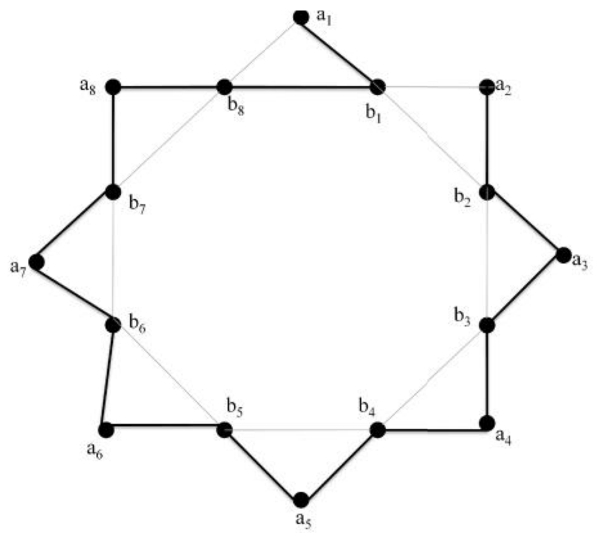

Figure 11: Hamiltonian path from the vertex a_1 to a_2 in Line graph $L[S_8]$

Lemma 2.2.2: The Line graph L(G), where $G=S_n$, is a Hamiltonian-t-laceability number, $(^{a}*(t))$ =1 for t=2 and 3 if even $n\geq 4$, where $1\leq t\leq$ diamG.*

Proof: Consider the graph $G=S_n$, its line $L(S_n)$. Here we need to establish the following cases to show that, Hamiltonian-t*-laceability number if $(^{a}*(t))$ =1 for t=2 and 3 if $n\geq 4$

Case (i): For t=2

In $L(S_n)$, we find that $d(a_1,b_2)=2$ and the path

$$P:(a_1,b_n)\cup(b_n,a_n)\cup(a_n,b_{n-1})\cup(b_{n-1},b_{n-2})\cup(b_{n-2},a_{n-2})\cup(a_{n-2},b_{n-3})\cup---\cup(b_{n-8},a_{n-8})\cup(a_{n-8},b_{n-9})\cup-----\cup(b_{n-11},b_{n-10})\cup(b_{n-10},a_{n-10})\cup(a_{n-10},b_{n-12})\cup--------\cup(b_4,a_4)\cup(a_4,b_3)\cup(b_3,a_3)\cup(a_3,a_2)\cup(a_2,b_1)\cup(b_1,b_2)$$

is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1,b_2)=2$. Therefore, G is a Hamiltonian-t*-laceable for t=2 and Laceability number $(^{a}*(t))$ =1 for t=2.

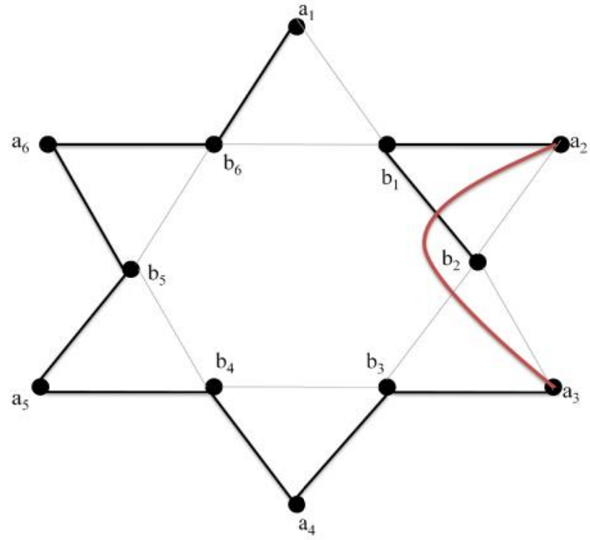

Figure 12: Hamiltonian path from the vertex a_1 to a_2 in Line graph $L[S_6]$

Case (ii): For t=3

In $L(S_n)$, we find that $d(a_1,b_3)=3$ and the path

$$P:(a_1,b_1)\cup(b_1,a_2)\cup(a_2,b_2)\cup(b_2,a_3)\cup(a_3,b_n)\cup(b_n,a_n)\cup(b_{n-1},a_{n-1})\cup(a_{n-1},b_{n-2})\cup-----\cup(a_6,b_5)\cup(b_5,a_5)\cup(a_5,b_4)\cup(b_4,a_4)\cup(a_4,a_3)\cup(a_3,b_3)$$

is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1,b_3)=3$. Therefore G is a Hamiltonian-t*-Laceability number $(^{a}*(t))$ =1 for t=3.

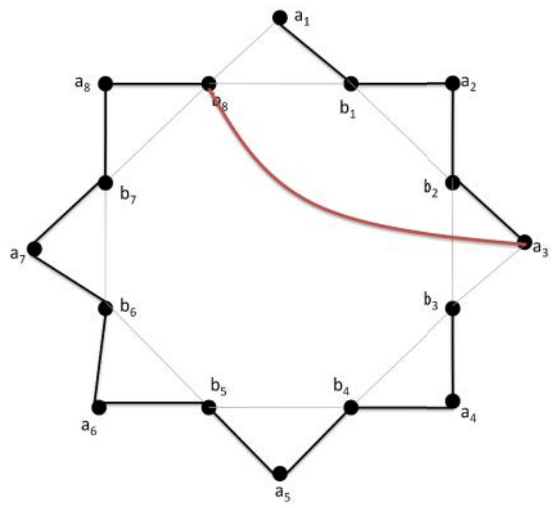

Figure 13: Hamiltonian path from the vertex a_1 to b_3 in Line graph $L[S_8]$

3. Remark

If n ≥ 4, the distance from $d(a_1,a_3)=3$ is a Hamiltonian-t*-laceable for t=3 and its laceability number $(a*(t))$ =1 for t=3, then the path

$$P:(a_1,b_1)\cup(b_1,b_n)\cup(b_n,a_n)\cup(a_n,b_{n-1})\cup(a_{n-1},b_{n-2})\cup------\cup(a_6,b_5)\cup(b_5,a_5)\cup(a_5,b_4)\cup(b_4,a_4)\cup(a_4,b_3)\cup(b_3,b_2)\cup(b_2,a_2)\cup(a_2,a_3)$$

is a Hamiltonian path

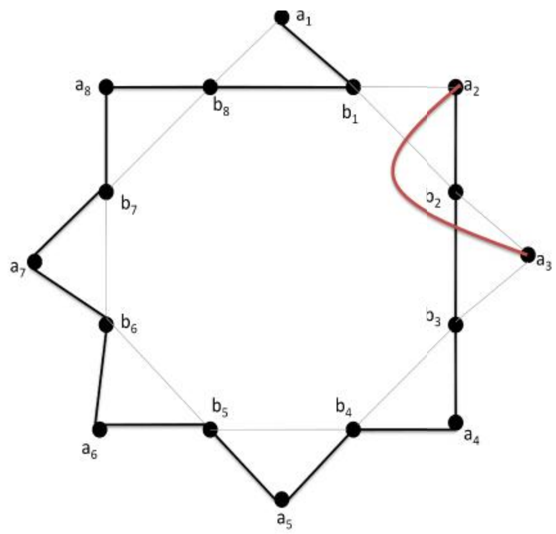

Figure 14: Hamiltonian path from the vertex a_1 to a_3 in Line graph $L[S_8]$

Theorem 2.3: The Line graph L (G), where $G=H_n$, n ≥ 3, the Helm graph is Hamiltonian-t*-laceable for t=1,2 and 3,with diameter 3.

Proof: Consider the graph $G=H_n$, its Line graph is denoted by $L(H_n)$ denote the vertices of L(G) by $a_1,b_1,c_1,\ a_2,b_2,c_2,\ a_3,b_3,c_3,\ a_4,b_4,c_4,\ -------,a_{n-1},b_{n-1},c_{n-1},\ a_n,b_n,c_n$. Hence we need to establish the following claims to show that G is a Hamiltonian-t*-laceable for t= 1,2 and 3 with diameter 3.

In Figure 15, we display the Helm graph H_n.

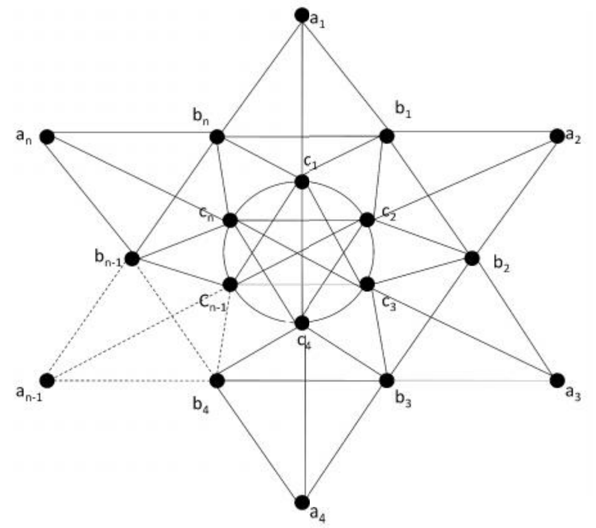

Figure 15

Claim 2.3.1: For t=1

Case (i): If n is odd

In $L(H_n)$, we find that $d(a_1,c_1)=1$ and the path

$$P:(a_1,b_1)\cup(b_1,b_n)\cup(b_n,c_n)\cup(c_n,a_n)\cup(a_n,b_{n-1})\cup(b_{n-1},c_{n-1})\cup(c_{n-1},a_{n-1})\cup(a_{n-1},b_{n-2})\cup(b_{n-2},c_{n-2})\cup(c_{n-2},a_{n-2})\cup(a_{n-2},b_{n-3})\cup(b_{n-3},c_{n-3})\cup(c_{n-3},a_{n-3})\cup----\cup(b_3,c_3)\cup(c_3,a_3)\cup(a_3,b_2)\cup(b_2,a_2)\cup(a_2,c_2)\cup(c_2,c_1)$$

is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1,c_1)=1$. Therefore G is a Hamiltonian-t*- Laceable for t=1.

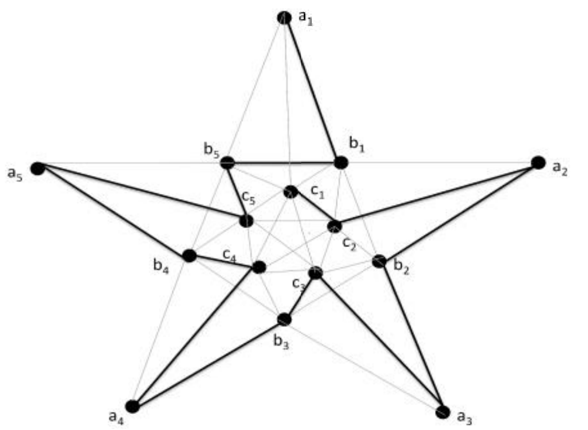

Figure 16: Hamiltonian path from the vertex a_1 to c_1 in Line graph $L[H_5]$

Case (ii): If n is even

In $L(H_n)$, we find that $d(a_1,c_1)=1$ and the path

$P:(a_1,b_n)\cup(b_n,a_n)\cup(a_n,c_n)\cup(c_n,b_{n-1})\cup$
$(b_{n-1},a_{n-1})\cup(a_{n-1},c_{n-1})\cup(c_{n-1},b_{n-2})\cup$
$(b_{n-2},a_{n-2})\cup----\cup(b_4,a_4)\cup(a_4,c_4)\cup$
$(c_4,b_4)\cup(b_4,b_3)\cup(b_3,a_3)\cup(a_3,c_3)\cup$
$(c_3,b_2)\cup(b_2,a_2)\cup(a_2,c_2)\cup(c_2,b_1)$
$\cup(b_2,c_1)$ is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1,c_1)=1$. Therefore G is a Hamiltonian-t*-Laceable for t=1.

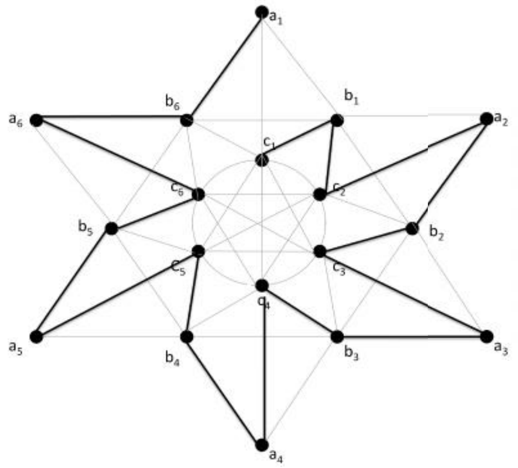

Figure 17: Hamiltonian path from the vertex a_1 to c_1 in Line graph L[H_6]

Claim 2.3.2: For t=2

Case (iii): If n is odd

In L(H_n), we find that $d(a_1,a_2)=2$ and the path

$P:(a_1,c_1)\cup(c_1,b_n)\cup(b_n,c_n)\cup(c_n,a_n)\cup$
$(a_n,b_{n-1})\cup(b_{n-1},c_{n-1})\cup(c_{n-1},a_{n-1})\cup$
$(a_{n-1},b_{n-2})\cup(b_{n-2},c_{n-2})\cup(c_{n-2},a_{n-2})\cup$
$-------\cup(c_{n-5},a_{n-5})\cup------\cup$
$(b_3,c_3)\cup(c_3,a_3)\cup(a_3,b_2)\cup(b_2,c_2)\cup$
$(c_2,b_1)\cup(b_1,a_2)$ is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1,a_2)=2$ Therefore G is a Hamiltonian-t*-Laceable for t=2.

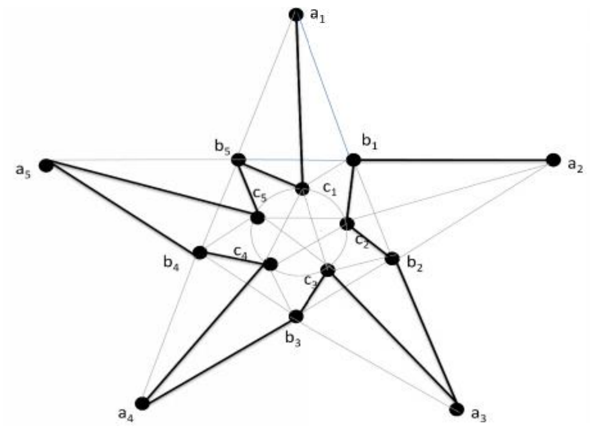

Figure 18: Hamiltonian path from the vertex a_1 to a_2 in Line graph L[H_5]

Case (iv): If n is even

In L(H_n), we find that $d(a_1,a_2)=2$ and the path

$P:(a_1,b_n)\cup(b_n,c_n)\cup(c_n,a_n)\cup(a_n,b_{n-1})\cup$
$(b_{n-1},c_{n-1})\cup(c_{n-1},a_{n-1})\cup(a_{n-1},b_{n-2})\cup$
$(b_{n-2},c_{n-2})\cup(c_{n-2},a_{n-2})\cup$
$----\cup(b_{n-4},a_{n-4})\cup------\cup(b_4,c_4)\cup$
$(c_4,a_4)\cup(a_4,b_3)\cup(b_3,c_3)\cup(c_3,a_3)\cup$
$(a_3,b_2)\cup(b_2,c_2)\cup(c_2,c_1)\cup(c_1,b_1)\cup$
(b_1,a_2)

is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1,a_2)=2$ d(a_1,a_2)=2. Therefore G is a Hamiltonian-t*-Laceable for t=2.

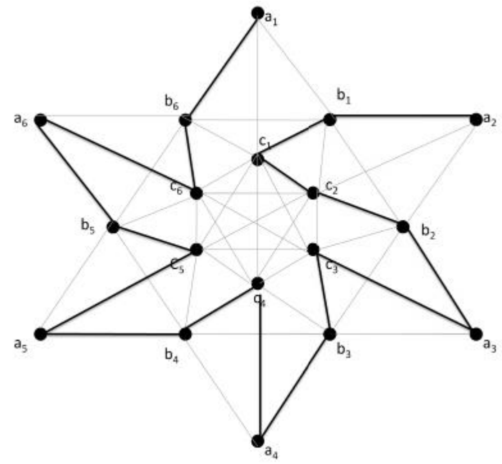

Figure 19: Hamiltonian path from the vertex a_1 to a_2 in Line graph L[H_6]

Claim 3: For t=3

Case (v): If n is odd

In L(H_n), we find that $d(a_1,a_3)=3$ and the path

$P:(a_1,b_1)\cup(b_1,b_n)\cup(b_n,c_n)\cup(c_n,a_n)\cup$
$(a_n,b_{n-1})\cup(b_{n-1},a_{n-1})\cup(a_{n-1},b_{n-2})\cup$

$(b_{n-2},c_{n-2}) \cup (c_{n-2},a_{n-2}) \cup (a_{n-2},b_{n-3}) \cup$
$----\cup (c_{n-12},a_{n-12}) \cup -----\cup (b_3,c_3) \cup$
$(c_3,c_2) \cup (c_2,a_2) \cup (a_2,b_2) \cup (b_2,a_3)$
is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1,a_3)=3$ d(a_1,a_3)=3. Therefore G is a Hamiltonian-t*-Laceable for t=3.

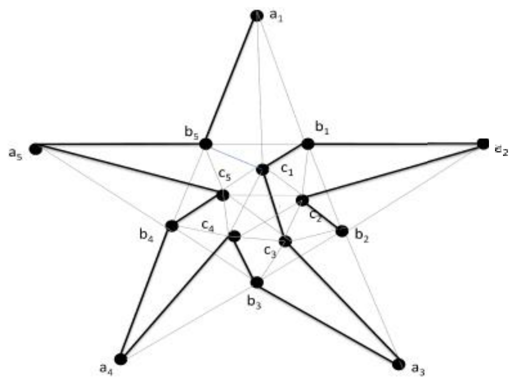

Figure 20: Hamiltonian path from the vertex a_1 to b_2 in Line graph L[H_5]

Case (vi): If n is even

In L(H_n), we find that $d(a_1,a_3)=3$ and the path

$P:(a_1,b_n) \cup (b_n,a_n) \cup (a_n,c_n) \cup (c_n,b_{n-1}) \cup$
$(b_{n-1},a_{n-1}) \cup (a_{n-1},c_{n-1}) \cup (c_{n-1},b_{n-2}) \cup$
$(b_{n-2},a_{n-2}) \cup (a_{n-2},c_{n-2}) \cup$
$----\cup (a_{n-7},c_{n-7}) \cup -----\cup (b_4,a_4) \cup$
$(a_4,c_4) \cup (c_4,b_5) \cup (b_5,a_5) \cup -------\cup$
$(c_2,b_1) \cup (b_1,c_1) \cup (c_1,c_3) \cup (c_3,a_3)$
is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_1,a_3)=3$. Therefore G is a Hamiltonian-t*- Laceable for t=3.

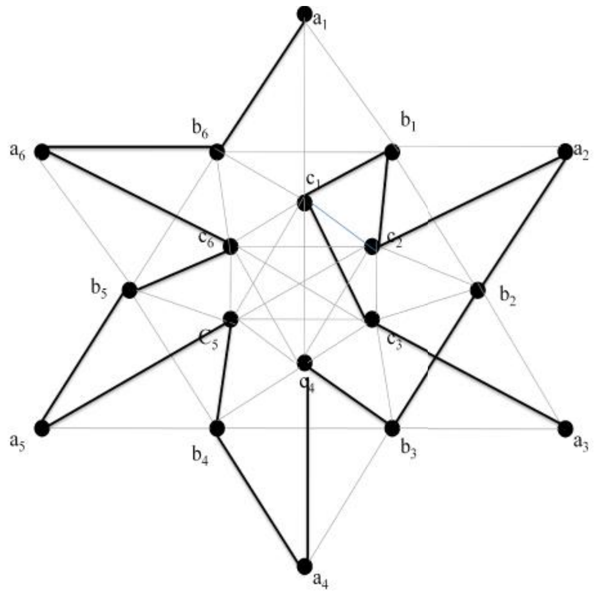

Figure 21: Hamiltonian path from the vertex a_1 to a_3 in Line graph L[H_6]

Theorem 2.4 The Line graph L (G), where G=G_n, n ≥ 4, the Gear graph is Hamiltonian-t*-laceable for t=1,2 and 3, with diameter 3

Proof: Consider the graph G=G_n, its Line graph is denoted by L(G_n) denote the vertices of L(G) by $a_1,a_2,a_3,a_4,-----,a_{n-1},a_n$. Hence we need to establish the following claims to show that G is a Hamiltonian-t*-laceable for t= 1,2 and 3 with diameter 3.

<u>Claim</u> 1: For t=1

Case (i): If n is odd

In L(G_n), we find that $d(a_0,a_1)=1$ and the path

$P:(a_0,a_{2n-2}) \cup (a_{2n-2},a_{3n-4}) \cup (a_{2n-3},a_{2n-4}) \cup$
$(a_{2n-9},a_{3n-9}) \cup -------\cup (a_{16},a_{15}) \cup$
$(a_{15},a_{2n+5}) \cup ----\cup (a_{14},a_{13}) \cup -----\cup$
$(a_6,a_{2n}) \cup (a_{2n},a_5) \cup (a_5,a_4) \cup (a_3,a_{2n-1}) \cup$
$(a_{2n-1},a_2) \cup (a_2,a_1)$ is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_0,a_1)=1$. Therefore G is a Hamiltonian-t*- Laceable for t=1.

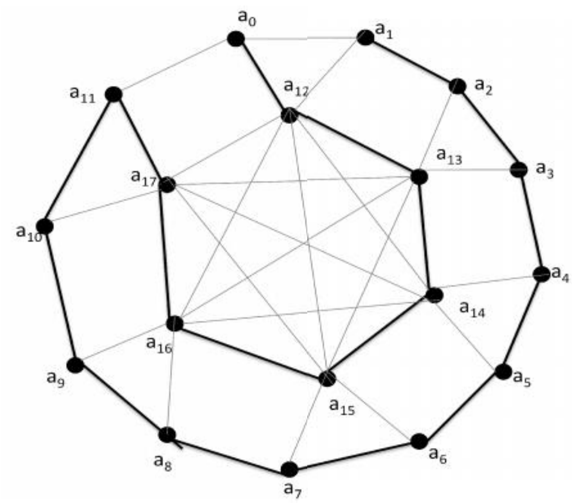

Figure 22: Hamiltonian path from the vertex a_0 to a_1 in Line graph L[G_7]

Case (ii): If n is even

In L(G_n), we find that $d(a_0,a_1)=1$ and the path

$P:(a_0,a_{2n-2}) \cup (a_{2n-2},a_{2n-1}) \cup (a_{2n-1},a_{2n}) \cup$
$(a_{2n},a_{2n+1}) \cup (a_{2n+1},a_{2n+2}) \cup (a_{2n+2},a_{2n+3}) \cup$
$----\cup (a_{15},a_{14}) \cup -----\cup (a_8,a_7) \cup$
$(a_7,a_6) \cup -----\cup (a_4,a_3) \cup (a_3,a_2) \cup$
(a_2,a_1) is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_0,a_1)=1$. Therefore G is a Hamiltonian-t*- Laceable for t=1.

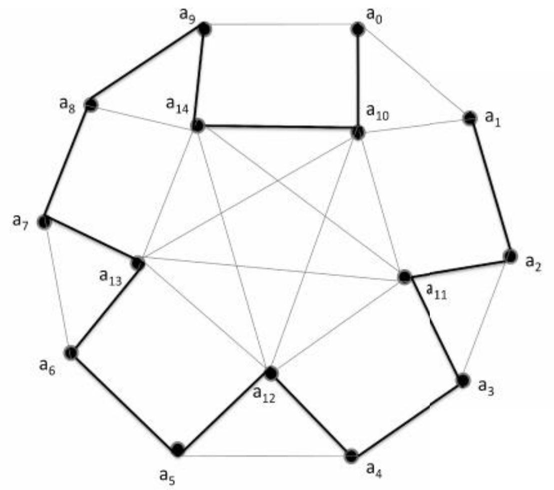

Figure 23: Hamiltonian path from the vertex a_0 to a_1 in Line graph L[G_6]

Claim 2.4.1: For t=2

Case (i): If n is odd

In L(G_n), we find that $d(a_0,a_2)=2$ and the path

$P:(a_0,a_1)\cup(a_1,a_{2n-2})\cup(a_{2n-2},a_{2n-1})\cup(a_{2n-1},a_{2n})\cup(a_{2n},a_{2n+1})\cup------------------\cup(a_{2n-3},a_{2n-4})\cup---------------\cup(a_{15},a_{14})\cup(a_{14},a_{13})\cup(a_{13},a_{12})\cup(a_{12},a_{11})\cup(a_{11},a_{10})\cup(a_{10},a_9)\cup(a_9,a_8)\cup(a_8,a_7)\cup(a_7,a_6)\cup(a_6,a_5)\cup(a_5,a_4)\cup(a_4,a_3)\cup(a_3,a_2)$ is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_0,a_2)=2$. Therefore G is a Hamiltonian-t*-Laceable for t=2

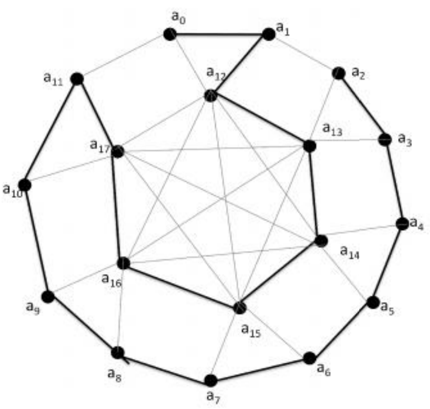

Figure 24: Hamiltonian path from the vertex a_0 to a_2 in Line graph L[G_7]

Case (ii): If n is even

In L(G_n), we find that $d(a_0,a_2)=2$ and the path

$P:(a_0,a_1)\cup(a_1,a_{2n-2})\cup(a_{2n-2},a_{2n-1})\cup(a_{2n-1},a_{2n})\cup(a_{2n},a_{2n+1})\cup------------------\cup(a_{2n-3},a_{2n-4})\cup---------------\cup(a_{15},a_{14})\cup(a_{14},a_{13})\cup(a_{13},a_{12})\cup(a_{12},a_{11})\cup(a_{11},a_{10})\cup(a_{10},a_9)\cup(a_9,a_8)\cup(a_8,a_7)\cup(a_7,a_6)\cup(a_6,a_5)\cup(a_5,a_4)\cup(a_4,a_3)\cup(a_3,a_2)$ is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_0,a_2)=2$. Therefore G is a Hamiltonian-t*-Laceable for t=2.

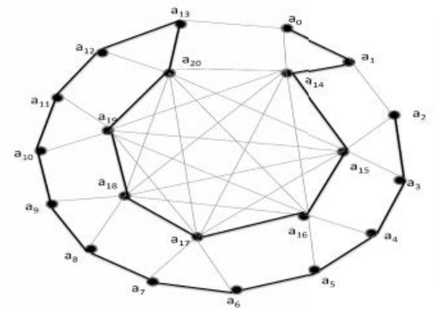

Figure 25: Hamiltonian path from the vertex a_0 to a_2 in Line graph L[G_8]

Claim 3.4.2: For t=3

Case (i): If n is odd

In L(G_n), we find that $d(a_0,a_3)=3$ and the path

$P:(a_0,a_1)\cup(a_1,a_2)\cup(a_2,a_{2n-1})\cup(a_{2n-1},a_{2n-2})\cup(a_{2n-2},a_{2n+7})\cup(a_{2n+7},a_{2n-3})\cup(a_{2n-3},a_{2n-4})\cup--------------------\cup(a_{14},a_{13})\cup----\cup(a_{2n+3},a_{11})\cup(a_{11},a_{10})\cup(a_{10},a_9)\cup----------\cup(a_{2n+2},a_{2n+3})\cup------\cup(a_7,a_6)\cup(a_6,a_5)\cup(a_5,a_{2n})\cup(a_{2n},a_4)\cup(a_4,a_3)$ is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_0,a_3)=3$. Therefore G is a Hamiltonian-t*- Laceable for t=3.

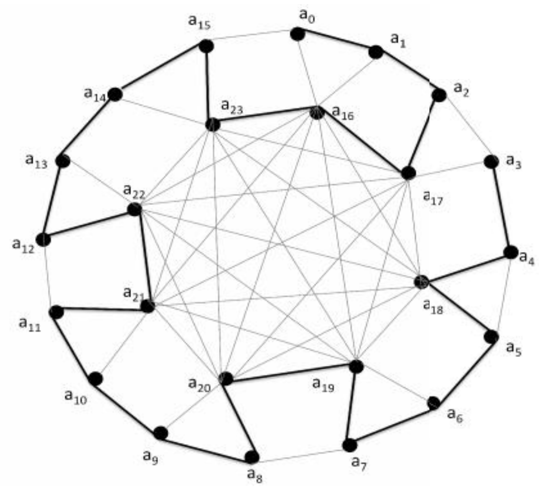

Figure 26: Hamiltonian path from the vertex a_0 to a_3 in Line graph L[G_9]

Case (ii): If n is even

In L(G_n), we find that $d(a_0, a_3) = 3$ and the path

$$P:(a_0,a_1)\cup(a_1,a_2)\cup(a_2,a_{2n-1})\cup(a_{2n-1},a_{2n-2})\cup(a_{2n-2},a_{3n-4})\cup(a_{3n-4},a_{2n-3})\cup(a_{19},a_{18})\cup(a_{18},a_{3n-5})\cup ------\cup(a_{17},a_{16})\cup(a_{16},a_{15})\cup(a_{15},a_{14})\cup(a_{14},a_{2n+5})\cup(a_{2n+5},a_{2n+4})\cup(a_{2n+4},a_{13})\cup ------\cup(a_{2n+3},a_{2n+2})\cup(a_9,a_8)\cup(a_8,a_7)\cup(a_7,a_6)\cup(a_6,a_{2n+1})\cup(a_{2n+1},a_{2n})\cup(a_{2n},a_5)\cup(a_5,a_4)\cup(a_4,a_3)$$

is a Hamiltonian path. Hence there exists a Hamiltonian path between at least one pair of vertices such that $d(a_0, a_3) = 3$. Therefore G is a Hamiltonian-t*- Laceable for t=3.

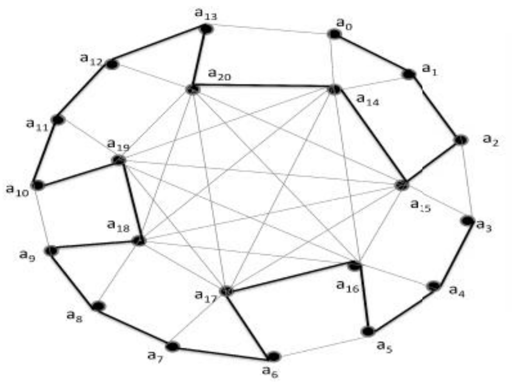

Figure 27: Hamiltonian path from the vertex a_0 to a_3 in Line graph L[G_8]

4. CONCLUSION

In this present study, the concept of Hamiltonian-t*-laceability in line graphs and t*-laceability number (are investigated. In our further work, Laceability of total graphs of other kind is to be proposed.

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