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18MAT21

Second Semester B.E. Degree Examination, June/July 2024
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the angle between surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, -1, 2). **(06 Marks)**

b. Evaluate div $\vec{F}$ and curl $\vec{F}$ for the vector point function $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ **(07 Marks)**

c. Determine the constants a, b and c so that the vector,

$$\vec{F} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$$

is irrotational and find ϕ such that $\vec{F} = \nabla\phi$ **(07 Marks)**

OR

2 a. If $\vec{F} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, evaluate $\int \vec{F}.d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path $x = t,\ y = t^2,\ z = t^3$. **(06 Marks)**

b. Using Green's theorem, evaluate $\int_C [(y - \sin x)dx + \cos x dy]$, where c is the plane triangle enclosed by the lines $y = 0,\ x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$ **(07 Marks)**

c. Evaluate $\int_s \vec{f}\cdot\hat{n}ds$ where $\vec{f} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and s is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$, by using the Gauss divergence theorem. **(07 Marks)**

Module-2

3 a. Solve : $4\frac{d^4y}{dx^4} - 4\frac{d^3y}{dx^3} - 23\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 36y = 0$. **(06 Marks)**

b. Find the solution of $(D^2 - 4D + 4)y = 8(e^{2x} + \sin 2x)$ by inverse operator method. **(07 Marks)**

c. Obtain the solution for the differential equation, $y'' - 2y' + y = \frac{e^x}{x}$ by the method of variation of parameter. **(07 Marks)**

OR

4 a. Find the solution for the differential equation $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = \sin 2x$. **(06 Marks)**

b. Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 9y = 3x^2 + \sin(3\log x)$ **(07 Marks)**

c. The differential equation of a simple pendulum is $\frac{d^2x}{dt^2}+\omega_0^2 x = F_0 \sin t$, where ω_0 and F_0 are constants. Solve it when $x = 0$, $\frac{dx}{dt}=0$ initially. **(07 Marks)**

Module-3

5 a. Form the partial differential equation by eliminating the arbitrary function f from $lx+my+nz=f(x^2+y^2+z^2)$ **(06 Marks)**

b. Solve by direct integration method.

$$\frac{\partial^3 z}{\partial x^2 \partial y}+18xy^2+\sin(2x-y)=0$$ **(07 Marks)**

c. Derive one dimensional heat equation in the form $\frac{\partial u}{\partial t}=C^2\frac{\partial^2 u}{\partial x^2}$. **(07 Marks)**

OR

6 a. Form the partial differential equation by eliminating the arbitrary function and from $f(x^2+y^2+z^2, z^2-2xy)=0$ **(06 Marks)**

b. Find the solution of the partial differential equation $(x^2-y^2-z^2)p+2xyq=2xz$ **(07 Marks)**

c. Obtain all possible solutions of one dimensional wave $\frac{\partial^2 U}{\partial t^2}=C^2\frac{\partial^2 U}{\partial x^2}$ by the method of variable separable method. **(07 Marks)**

Module-4

7 a. Test the convergence of the series,

$$\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+\dots\infty$$ **(06 Marks)**

b. Obtain the series solution of Bessel's differential equation which leads to $J_n(x)$. **(07 Marks)**

c. Express the polynomial $f(x)=x^4+3x^3-x^2+5x-2$ in terms of Legendre polynomials. **(07 Marks)**

OR

8 a. Test for convergence for, $\sum\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2}}}$. **(06 Marks)**

b. Prove that $\int_0^1 xJ_n(\alpha x).J_n(\beta x)dx=0$ for $\alpha \neq \beta$ and α, β are the roots of the equation $J_n(x)=0$. **(07 Marks)**

c. Using the Rodrigue's formula, find the Legendre polynomials $P_0(x)$, $P_1(x)$, $P_2(x)$ and $P_3(x)$. **(07 Marks)**

Module-5

9 a. Compute the real root of $x\log_{10}x-1.2=0$ by the method of false position. Correct to 3 decimal places, which lies between 2 and 3. **(06 Marks)**

b. Using the Newton's divided difference method find the interpolating polynomial of the given data :

x	-1	0	1	3
f(x)	2	1	0	-1

(07 Marks)

c. By using Simpson's $\frac{1}{3}$ rule, evaluate $\int_0^6 \frac{dx}{1+x^2}$ by considering seven ordinates. **(07 Marks)**

OR

10 a. Use Newton-Raphson method to find a real root of $x \sin x + \cos x = 0$ near $x = \pi$ correct to three decimal places. **(06 Marks)**

b. From the following table, estimate the number of students who obtained marks between 40 and 45. **(07 Marks)**

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Number of students	31	42	51	35	31

c. Evaluate $\int_4^{5.2} \log x \, dx$, by using the Weddle's Rule taking Seven ordinates. **(07 Marks)**

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