# CBCS SCHEME

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**BMATM201**

## Second Semester B.E./B.Tech. Degree Examination, June/July 2024
## Mathematics – II for Mechanical Engineering Stream

Time: 3 hrs. Max. Marks: 100

*Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.*
*2. M : Marks , L: Bloom's level , C: Course outcomes.*
*3. Mathematics hand book is permitted.*

| | | Module – 1 | M | L | C |
|---|---|---|---|---|---|
| Q.1 | a. | Evaluate $\int_0^a\int_0^b\int_0^c (x^2+y^2+z^2)dxdydz$. | 6 | L2 | CO1 |
| | b. | Change the order of integration in $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dydx$ and hence evaluate the same. | 7 | L2 | CO1 |
| | c. | Derive the relation between Beta and Gamma function. | 7 | L2 | CO1 |
| | | **OR** | | | |
| Q.2 | a. | Evaluate $\int_0^1\int_0^{\sqrt{1-y^2}} (x^2+y^2)dxdy$ by changing to polar coordinates. | 7 | L2 | CO1 |
| | b. | Using double integration, find the volume of the tetrahedron bounded by the planes x = 0, y = 0, z = 0 and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. | 7 | L2 | CO1 |
| | c. | Write a modern mathematical program to evaluate the integral $\int_0^3\int_0^{3-x}\int_0^{3-x-y} xyz\,dz\,dy\,dx$. | 6 | L3 | CO5 |
| | | **Module – 2** | | | |
| Q.3 | a. | If $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$ then show that $\nabla r^n = nr^{n-2}\vec{r}$. | 7 | L2 | CO2 |
| | b. | Find the directional derivative of $\phi = x^2yz+4xz^2$ at (1, -2, -1) along $2\hat{i}-\hat{j}-2\hat{k}$. | 7 | L2 | CO2 |
| | c. | If $\vec{F} = (x+y+az)\hat{i}+(bx+2y-z)\hat{j}+(x+cy+2z)\hat{k}$, find a, b, c such that curl $\vec{F} = \vec{0}$. | 6 | L2 | CO2 |
| | | **OR** | | | |
| Q.4 | a. | Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{i}+(2xz-y)\hat{j}+z\hat{k}$ along the straight line from (0, 0, 0) to (2, 1, 3). | 7 | L3 | CO2 |

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<tr><td></td><td>b.</td><td>Using Green's theorem, evaluate $\int_c (xy + y^2)dx + x^2dy$, where C is the closed curve of the region bounded by y = x and y = $x^2$.</td><td>7</td><td>L2</td><td>CO2</td></tr>
<tr><td></td><td>c.</td><td>Write a modern mathematical tool program to find the divergence of $\vec{F} = x^2yz\hat{i} + y^2zx\hat{j} + z^2xy\hat{k}$.</td><td>6</td><td>L3</td><td>CO2</td></tr>
<tr><td colspan="6"><b>Module – 3</b></td></tr>
<tr><td>Q.5</td><td>a.</td><td>Form the partial differential equation from the relation<br>z = f(y + 2x) + g(y – 3x).</td><td>6</td><td>L1</td><td>CO3</td></tr>
<tr><td></td><td>b.</td><td>Solve $\frac{\partial^2 z}{\partial x^2} = xy$ subject to the conditions $\frac{\partial z}{\partial x} = \log(1 + y)$ when x = 1 and z = 0 when x = 0.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c.</td><td>Derive one dimensional heat equation.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6"><b>OR</b></td></tr>
<tr><td>Q.6</td><td>a.</td><td>Form the partial differential equation from the relation<br>$f(xy + z^2, x + y + z) = 0$</td><td>6</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>b.</td><td>Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that when x = 0, z = $e^y$ and $\frac{\partial z}{\partial x} = 1$.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c.</td><td>Solve $(mz – ny)p + (nx – \ell z)q = \ell y – mx$.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6"><b>Module – 4</b></td></tr>
<tr><td>Q.7</td><td>a.</td><td>Find the real root of the equation $\cos x – xe^x = 0$ in (0.5, 0.6) using the Regula – Falsi method correct to four decimal places, carryout there interations.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>The population of a town is given by the table
<table>
<tr><td>Year</td><td>1951</td><td>1961</td><td>1971</td><td>1981</td><td>1991</td></tr>
<tr><td>Population in thousands</td><td>19.96</td><td>39.65</td><td>58.81</td><td>77.21</td><td>94.61</td></tr>
</table>
Using Newton's forward interpolation formula, calculate the population in the year 1955.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's 1/3rd rule. [Take 6 equal parts].</td><td>6</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6"><b>OR</b></td></tr>
<tr><td>Q.8</td><td>a.</td><td>Find a real root of the equation $x^3 + 5x – 11 = 0$ near to x = 1 using Newton – Raphson method. Carryout three iterations.</td><td>7</td><td>L3</td><td>CO4</td></tr>
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<tr><td></td><td><b>b.</b></td><td>Using Newton's divided difference formula, evaluate f(4) from the following table<table><tr><td>x</td><td>0</td><td>2</td><td>3</td><td>6</td></tr><tr><td>f(x)</td><td>-4</td><td>2</td><td>14</td><td>158</td></tr></table></td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td><b>c.</b></td><td>Evaluate $\int_0^1 \frac{dx}{1+x}$ taking seven ordinates by applying Simpson's (3/8)th rule.</td><td>6</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6"><b>Module – 5</b></td></tr>
<tr><td><b>Q.9</b></td><td><b>a.</b></td><td>Using Taylor's series method, find y(0.1) considering upto fourth degree term if y(x) satisfies the equation $\frac{dy}{dx} = x - y^2, y(0) = 1.$</td><td>6</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td><b>b.</b></td><td>Using Runge-Kutta method of fourth order, find y(0.2) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0) = 1 taking h = 0.2.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td><b>c.</b></td><td>Given that $\frac{dy}{dx} = x - y^2$ and the data y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762. Compute y at x = 0.8 applying Milne's method.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6"><b>OR</b></td></tr>
<tr><td><b>Q.10</b></td><td><b>a.</b></td><td>Using modified Euler's method, compute y(1.1) given that $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ and y = 1 at x = 1 by taking h = 0.1.</td><td>7</td><td>L3</td><td>CO5</td></tr>
<tr><td></td><td><b>b.</b></td><td>Apply Runge-Kutta fourth order method, to find an approximate value of y when x = 0.2, given that $\frac{dy}{dx} = x + y$ and y = 1 when x = 0.</td><td>7</td><td>L3</td><td>CO5</td></tr>
<tr><td></td><td><b>c.</b></td><td>Using modern mathematical tools with a program to find y when x = 1.4, given $\frac{dy}{dx} = x^2 + (y/2), y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649,$ y(1.3) = 2.7514 using predictor corrector method.</td><td>6</td><td>L3</td><td>CO5</td></tr>
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