CBCS SCHEME

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BMATC101

# First Semester B.E./B.Tech. Degree Examination, June/July 2024
## Mathematics – I for Civil Engineering Stream

Time: 3 hrs. Max. Marks: 100

*Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.*
*2. VTU Formula Hand Book is permitted.*
*3. M : Marks , L: Bloom's level , C: Course outcomes.*

| | | Module – 1 | M | L | C |
|---|---|---|---|---|---|
| Q.1 | a. | Derive an expression for the radius of curvature for Cartesian curve $y = f(x)$. | 6 | L1 | CO1 |
| | b. | Show that the pedal equation of the curve $r^m \cos m\theta = a^m$ is $pr^{m-1} = a^m$. | 7 | L3 | CO1 |
| | c. | Show that the radius of curvature at any point of the Cardiod $r = a(1-\cos\theta)$ varies as $\sqrt{r}$. | 7 | L3 | CO1 |
| | | **OR** | | | |
| Q.2 | a. | With usual notations, prove that for the curve, $r = f(\theta)$, $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$ | 8 | L3 | CO1 |
| | b. | For the curve $y = \frac{ax}{a+x}$, show that $\left(\frac{2\rho}{a}\right)^{\frac{2}{3}} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$. | 7 | L3 | CO1 |
| | c. | Using modern mathematical tool, write a programme to plot the sine and cosine curve. | 5 | L3 | CO5 |
| | | **Module – 2** | | | |
| Q.3 | a. | Evaluate $\lim_{x\to 0}\left(\frac{1^x + 2^x + 3^x}{3}\right)^{\frac{1}{x}}$. | 6 | L1 | CO2 |
| | b. | If $u = e^{ax+by} f(ax-by)$, prove that $b\frac{\partial u}{\partial x} + a\frac{\partial u}{\partial y} = 2abu$ | 7 | L2 | CO2 |
| | c. | If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. | 7 | L2 | CO2 |
| | | **OR** | | | |
| Q.4 | a. | If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ find the value of $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z}$. | 7 | L1 | CO2 |
| | b. | Show that $f(x,y) = x^3 + y^3 - 3xy + 1$ is minimum at (1, 1). | 7 | L3 | CO2 |
| | c. | Using modern mathematical tool, write a programme to show that $u_{xx} + u_{yy} = 0$, given $u = e^x[x\cos y - y\sin y]$. | 6 | L3 | CO5 |
| | | **Module – 3** | | | |
| Q.5 | a. | Solve : $(1-x^2)\frac{dy}{dx} - xy = 1$. | 6 | L3 | CO3 |
| | b. | Find the orthogonal trajectories of the family of confocal and coaxial parabolas $r = \frac{2a}{1+\cos\theta}$. | 7 | L1 | CO3 |
| | c. | Solve $yp^2 + (x-y)p - x = 0$ | 7 | L2 | CO3 |

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| | | **OR** | | | |
| **Q.6** | **a.** | Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$. | **6** | **L3** | **CO3** |
| | **b.** | A metal ball is heated to a temperature of 100°C and at time t = 0 it is placed in heater which is maintained at 40°C. If temperature of the ball is reduced to 60°C in 4 min, find the time at which the temperature of the ball is 50°C. | **7** | **L2** | **CO3** |
| | **c.** | Find the general and singular solution of $(px - y)(x - py) = 2p$ by using the substitution $x^2 = u$ and $y^2 = v$. | **7** | **L2** | **CO3** |
| | | **Module – 4** | | | |
| **Q.7** | **a.** | Solve : $(D^2 - D + 2)y = \sin 2x$. | **6** | **L3** | **CO4** |
| | **b.** | Solve by the method of variation of parameters $y'' - 2y' + y = e^x \log x$. | **7** | **L3** | **CO4** |
| | **c.** | Solve : $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$. | **7** | **L3** | **CO4** |
| | | **OR** | | | |
| **Q.8** | **a.** | Solve : $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$. | **6** | **L3** | **CO4** |
| | **b.** | Find the complete solution of. $(D^2 - 2D + 2)y = x + e^x$. | **7** | **L3** | **CO4** |
| | **c.** | Solve $(1 + x^2)\frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + y = \sin(2\log(1 + x))$ | **7** | **L3** | **CO4** |
| | | **Module – 5** | | | |
| **Q.9** | **a.** | Find the rank of the matrix, $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & -3 & 1 & 2 \\ -3 & -4 & 5 & 8 \\ 1 & 3 & 10 & 14 \end{bmatrix}$ | **6** | **L1** | **CO4** |
| | **b.** | Investigate the values of $\lambda$ and $\mu$ so that the equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have, (i) No solution (ii) Unique solution (iii) Infinite solution | **7** | **L2** | **CO4** |
| | **c.** | Using the Rayleigh's power method, find the largest Eigen value and the corresponding Eigen vector of the matrix, $A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 8 & 5 \end{bmatrix}$ by taking $[1 \quad 0 \quad 0]^T$ as the initial Eigen vector perform four iterations. | **7** | **L2** | **CO4** |
| | | **OR** | | | |
| **Q.10** | **a.** | Apply Gauss-Seidal iterative method to solve the equations $10x + y + z = 12$, $x + 10y + z = 12$, $x + y + 10z = 12$ by taking the initial approximation to the solution as, (0, 0, 0) (Carry out 4 iterations). | **7** | **L2** | **CO4** |
| | **b.** | Solve the system of equations by Gauss Elimination method. $x + y + z = 6$, $x - y + 2z = 5$, $3x + y + z = 8$ | **7** | **L3** | **CO4** |
| | **c.** | Using modern mathematical tool write a programme to test the consistency of the equation $x + 2y - z = 1$, $2x + y + 4z = 2$, $3x + 3y + 4z = 1$ | **6** | **L3** | **CO5** |

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