CBCS SCHEME

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18MAT11

First Semester B.E. Degree Examination, June/July 2024
Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find the angle between the curves, $r = a(1 + \sin\theta)$ and $r = a(1 - \sin\theta)$. **(08 Marks)**

b. Show that for the curve $r(1 - \cos\theta) = 2a$, ρ^2 varies as r^3. **(06 Marks)**

c. Derive $\tan\phi = r\dfrac{d\theta}{dr}$ with usual notation. **(06 Marks)**

OR

2 a. Find the pedal equation of the curve $r^2 = a^2[\cos 2\theta + \sin 2\theta]$. **(08 Marks)**

b. If ρ be the radius of curvature at any point P(x, y) on $y^2 = 4ax$, show that $a\rho^2 = 4(x + a)^3$. **(06 Marks)**

c. Show that the evolute of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is $(xa)^{2/3} + (yb)^{2/3} = (a^2 - b^2)^{2/3}$. **(06 Marks)**

Module-2

3 a. Expand $\log[1 + e^x]$ using Maclaurin's series upto the term containing x^3. **(06 Marks)**

b. Evaluate : $\underset{x \to 0}{\ell t}\left[\dfrac{3^x + 4^x + 5^x}{3}\right]^{1/x}$. **(07 Marks)**

c. If $z = f(x, y)$ with $x = r\cos\theta$ and $y = r\sin\theta$ show that :

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2.$$

(07 Marks)

OR

4 a. If $U = f(x - y, y - z, z - x)$ prove that $\dfrac{\partial U}{\partial x} + \dfrac{\partial U}{\partial y} + \dfrac{\partial U}{\partial z} = 0$. **(06 Marks)**

b. Find the Jacobian, $\dfrac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$ from $x = r\cos\theta\cos\phi$, $y = r\cos\theta\sin\phi$ and $z = r\sin\theta$. **(07 Marks)**

c. Show that the rectangular box of maximum volume and a given surface area is cube. **(07 Marks)**

Module-3

5 a. Evaluate : $\iint_R x^2 y\,dx\,dy$, over the region bounded by the curves $y = x^2$ and $y = x$. **(06 Marks)**

b. Find the volume generated by the revolution of the cardioide $r = a(1 + \cos\theta)$ about the initial line, using double integral. **(07 Marks)**

c. Using definition of Gamma function, show that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$. **(07 Marks)**

OR

6 a. Evaluate : $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\,dy\,dx$ by changing into polar co-ordinates. **(06 Marks)**

b. Evaluate : $\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$. **(07 Marks)**

c. Show that : $\int_0^\infty \sqrt{x}\, e^{-x^2} dx \times \int_0^\infty \frac{e^{-x^2}}{\sqrt{x}} dx = \frac{\pi}{2\sqrt{2}}$. **(07 Marks)**

Module-4

7 a. A body in air at 25°C cools from 100°C to 75°C in 1 minute. Find the temperature of the body at the end of 3 minutes. **(06 Marks)**

b. Find the orthogonal trajectories of the family of curves $r^n = a^n \sin n\theta$, where 'a' is a parameter. **(07 Marks)**

c. Solve : $y\left(\frac{dy}{dx}\right)^2 + (x-y)\frac{dy}{dx} - x = 0, p = \frac{dy}{dx}$. **(07 Marks)**

OR

8 a. Solve : $(2x^2 - 6xy)dy + (8xy - 9y^2)dx = 0$. **(06 Marks)**

b. Solve : $\frac{dy}{dx} + y^2 \tan x = y^3 \sec x$. **(07 Marks)**

c. The current 'i' in an electrical circuit containing an inductance L and a resistance R in series and, acted upon an e.m.f $E\sin\omega t$ satisfies the differential equation :

$$L\frac{di}{dt} + R.i = E\sin\omega t$$

Find the value of the current at any time, if initially there is no current in the circuit. **(07 Marks)**

Module-5

9 a. By applying elementary row operations find rank of :

$$\begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 2 \\ 3 & 2 & 2 & 3 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

(06 Marks)

b. Solve using Gauss – Seidel method [carry out 4 iterations] :

$$6x + 15y + 2z = 72$$
$$27x + 6y - z = 85$$
$$x + y + 54z = 110.$$

(07 Marks)

c. Diagonalize the square matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. **(07 Marks)**

OR

10 a. Apply Gauss – Jordan method to solve the following system of equations :

$$2x + y + 3z = 1$$
$$4x + 4y + 7z = 1$$
$$2x + 5y + 9z = 3.$$

(06 Marks)

b. Test for consistency, if consistent solve it.

$$x + 2y + 3z = 14$$
$$4x + 5y + 7z = 35$$
$$3x + 3y + 4z = 21.$$

(07 Marks)

c. Using Rayleigh's power method, find largest eigen value and the corresponding eigen vector of :

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

By taking $X^{(0)} = [1, 1, 1]^T$ as initial eigen vector. **(07 Marks)**

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