CBCS SCHEME

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18AI56

Fifth Semester B.E. Degree Examination, June/July 2024
Mathematic for Machine Learning

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find all solutions of the inhomogeneous system of linear equations Ax = b where

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ -1 & 2 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$ **(07 Marks)**

b. Find the image and kernel of a linear. Mapping

$$\phi: R^4 \to R^2, \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$ **(07 Marks)**

c. Consider R^3 with <·, ·> defined for all x, y ∈ R^3 as <x, y> = x^TAy, $A = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 4 & -1 \\ 1 & -1 & 5 \end{bmatrix}$

gs <·, ·> an inner product? **(06 Marks)**

OR

2 a. Find all solutions of system of equations :

$$-2x_1 + 4x_2 - 2x_3 - x_4 + 4x_5 = -3$$
$$4x_1 - 8x_2 + 3x_3 - 3x_4 + x_5 = 2$$
$$x_1 - 2x_2 + x_3 - x_4 + x_5 = 0$$
$$x_1 - 2x_2 - 3x_4 + 4x_5 = a.$$ **(07 Marks)**

b. Show that the vectors $\alpha_1 = (1, 0, -1)$ $\alpha_2 = (1, 2, 1)$, $\alpha_3 = (0, -3, 2)$ form a basis for R^3. Express each of the standard basis vector Q linear combination of α_1, α_2, α_3. **(07 Marks)**

c. Define an inner product space. For any vector α. β in an inner product space V prove that $||\alpha + \beta|| \le ||\alpha|| + ||\beta||$. **(06 Marks)**

Module-2

3 a. For a subspace $U = \text{span}\left\{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}\right\} \le R^3$ and $x = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix} \in R^3$ find the coordinates λ of x in terms of the subspace U, the projection point $\pi_U(x)$ and the projection matrix P_π. **(10 Marks)**

b. Diagonalize the matrix $A = \begin{bmatrix} 4 & 0 & -2 \\ 1 & 3 & -2 \\ 1 & 2 & -1 \end{bmatrix}$. **(10 Marks)**

OR

4 a. Apply gram Schmidt orthogonalization process to the basis B = (1, 1, 1), (–1, 0, 1), (–1, 2, 3)} of the inner product space R^3 to find an orthogonal basis of R^3. Also find orthogonal basis of R^3. **(10 Marks)**

b. Find singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$. **(10 Marks)**

Module-3

5 a. Compute the partial derivative $\frac{\partial f}{\partial A}$ for the function f = Ax where $A \in R^{3x2}$ and $x \in R^2$. **(07 Marks)**

b. Consider $f(x_1, x_2) = x_1^2 + 2x_2$ where $x_1 = \sin t$ and $x_2 = \cos t$. find derivative of f with respect to t. **(06 Marks)**

c. Obtain the gradient $\frac{df}{dx}$ for the function f(x) = Ax, $f(x) \in R^M$, $A \in R^{MXN}$, $x \in R^N$. **(07 Marks)**

OR

6 a. Consider the linear model $y = \phi\ \theta$ wher $\theta \in R^D$ is a parameter vector, $\phi \in R^{NXD}$ are input features and $y \in R^N$ are corresponding observation we define least squares loss function :

$L(e) : || e ||^2, e(\theta) ; y - \phi\theta$. Find $\frac{\partial L}{\partial \theta}$. **(06 Marks)**

b. For the function $f(x) = \sqrt{x^2 + \exp(x^2) + \cos(x^2 + \exp(x^2))}$ find $\frac{\partial f}{\partial x}$. **(07 Marks)**

c. Consider the matrix $R \in \mathbb{R}^{MXN}$ and $f : \mathbb{R}^{MXN} \rightarrow \mathbb{R}^{NXN}$ with $f(R) = R^T R = K \in R^{NXN}$ find gradient dK/dR. **(07 Marks)**

Module-4

7 a. The probability that the noise level of a wide band amplifier will exceed 2dB is 0.05. Find the probabilities that among 12 such amplifiers the noise level of :

i) One will exceed 2dB
ii) Atmost 2 will exceed 2dB
iii) Two or more will exceed 2dB. **(06 Marks)**

b. Let X_1 and X_2 have the joint probability distribution :

x_2 \ x_1	0	1	2
0	0.1	0.4	0.1
1	0.2	0.2	0

i) Find marginal distribution of x_1 and x_2
ii) Find $P(x_1 + x_2 > 1)$
iii) Find conditional probability distribution of x_1 given $x_2 = 1$. And x_1 and x_2 are Independent. **(07 Marks)**

c. If x is a Poisson variate such that P(x = 2) = 9P(x = 4) + 90 P (X = 6). Find mean of x. **(07 Marks)**

OR

8 a. The probabilities of X, Y, Z becoming manager are 4/9, 2/9 and 1/3 respectively. The probabilities that the bonus scheme will be introduced if X, Y, Z become managers are 3/10, 1/2, 4/5 respectively. **(06 Marks)**

i) What is the probability that bonus will be introduced

ii) If the bonus scheme is introduced, what is the probability that manger appointed is X?

b. Verify that the function P(x) defined by

$$P(x) = \begin{cases} e^{-x} & \text{for} \quad x \geq 0 \\ 0 & \text{for} \quad x < 0 \end{cases}$$

is a probability density function. Find the probability that variable x having this density falls in the interval (1.5 , 2.5). Also evaluate cumulative distribution function F(2.5). **(07 Marks)**

c. Let n random variables $X_1, X_2, \ldots\ldots, X_n$ be independent and each have the same distribution with mean μ and variance σ^2. Use the properties of expectation to show that the sample mean $\overline{X}$ has i) mean $\mu_{\bar{x}} = E(\overline{X}) = \mu$ ii) Variance $\sigma_{\bar{x}}^2 = \text{Var}(\overline{X}) = \frac{\sigma^2}{n}$. **(07 Marks)**

Module-5

9 a. Using Lagrange's multiplier method, find the stationary value of the function $f(x, y, z) = x^2 y^2 z^2$ subject to the conditions $x^2 + y^2 + z^2 = a^2$. **(07 Marks)**

b. Check whether the function $f(x) = x \log_2^x$ is convex or not. **(07 Marks)**

c. Derive the dual linear program using Lagrange duality for the linear program $\min_{x \in R^d} C^T x$, subject to $Ax \leq b$, where $A \in R^{m \times d}$, $b \in R^m$ and $C \in R^d$. **(06 Marks)**

OR

10 a. Find local minimum using gradient descent for the function $f(x) = x_1^2 - 2x_1 x_2 + 2x_2^2 + 2x_1$. **(07 Marks)**

b. Given x + y + z = a, find the maximum value of $x^m y^n z^p$. **(07 Marks)**

c. If f_1 and f_2 are two convex functions then show that $\alpha f_1(x) + \beta f_2(x)$ is also a convex function. **(06 Marks)**

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