USN | | | | | | | | | |

Third Semester B.E. Degree Examination, Dec.2023/Jan.2024
Additional Mathematics – I

Time: 3 hrs.　　　　　　　　　　　　　　　　　　　Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1　a.　Show that
$$\frac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^4} = \cos 8\theta + i\sin 8\theta$$
(05 Marks)

　　b.　Prove that $[\vec{b}\times\vec{c},\ \vec{c}\times\vec{a},\ \vec{a}\times\vec{b}] = [\vec{a}\ \ \vec{b}\ \ \vec{c}]^2$
(05 Marks)

　　c.　Prove that $\hat{i}\times(\vec{a}\times\hat{i}) + \hat{j}\times(\vec{a}\times\hat{j}) + \hat{k}\times(\vec{a}\times\hat{k}) = 2\vec{a}$
(06 Marks)

OR

2　a.　Find the real part of $\dfrac{1}{1+\cos\theta + i\sin\theta}$
(05 Marks)

　　b.　If $\vec{a} = 3i - 2j + 4k$ and $\vec{b} = i + j - 2k$, find (i) $\vec{a}.\vec{b}$　(ii) Angle between $\vec{a}$ & $\vec{b}$.
(05 Marks)

　　c.　Show that $[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}] = 2[a,b,c]$
(06 Marks)

Module-2

3　a.　If $y = \tan^{-1}x$, prove that $(1+x^2)y_2 + 2xy_1 = 0$ and hence show that
$(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$
(06 Marks)

　　b.　Find the angle between the radius vector and the tangent for the curve
$r^m = a^m(\cos m\theta + \sin m\theta)$
(05 Marks)

　　c.　Find the pedal equation of $r^n = a^n \cos n\theta$
(05 Marks)

OR

4　a.　If $u = \log\left(\dfrac{x^4 + y^4}{x+y}\right)$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3$
(06 Marks)

　　b.　If $u = f(x-y,\ y-z,\ z-x)$, show that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$
(05 Marks)

　　c.　Find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ where $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x+y+z$
(05 Marks)

Module-3

5　a.　Evaluate the integral $\displaystyle\int_0^\pi x\sin^2 x\cos^4 x\,dx$　using Reduction formula.
(05 Marks)

　　b.　Evaluate $\displaystyle\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$
(05 Marks)

　　c.　Evaluate $\displaystyle\int_{-1}^1 \int_0^2 \int_{x-2}^{x+2} (x+y+z)\,dy\,dx\,dz$
(06 Marks)

OR

6   a.   Evaluate $\int_0^\pi \sin^4 x \ dx$, using Reduction formula. **(05 Marks)**

    b.   Evaluate $\int_0^1 x^{3/2}(1-x)^{3/2} \ dx$, using Reduction formula. **(06 Marks)**

    c.   Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) \ dy \ dx$, using Reduction formula. **(05 Marks)**

Module-4

7   a.   A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$ where t is the time. Find the velocity and acceleration at any time t and also their magnitudes at t = 0. **(05 Marks)**

    b.   Find $\operatorname{div} \vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ **(06 Marks)**

    c.   Find the value of the constant 'a' such that the vector field $\vec{F} = (axy - z^3)i + (a-2)x^2 j + (1-a)xz^2 k$ is irrotational. **(05 Marks)**

OR

8   a.   If $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ represents the parametric equation of the curve. Find the angle between the tangents at t = 1 and t = 2. **(05 Marks)**

    b.   Find the angle between the normals to the surface at the points (4, 1, 2) and (3, 3, −3). **(05 Marks)**

    c.   Show that $\vec{F} = \dfrac{xi + yj}{x^2 + y^2}$ is both solenoidal and irrotational. **(06 Marks)**

Module-5

9   a.   Solve : $x^2 y \ dx - (x^3 + y^3) dy = 0$ **(06 Marks)**

    b.   Solve : $(2x + y + 1)dx + (x + 2y + 1)dy = 0$ **(05 Marks)**

    c.   Show that $\dfrac{dy}{dx} - \dfrac{2y}{x} = x + x^2$ **(05 Marks)**

OR

10 a.   Solve : $(y^3 - 3x^2 y)dx - (x^3 - 3xy^2)dy = 0$ **(05 Marks)**

    b.   Solve : $\dfrac{dy}{dx} + y\cot x = \cos x$ **(05 Marks)**

    c.   Solve : $\dfrac{dy}{dx} + \dfrac{y}{x} = y^2 x$ **(06 Marks)**

* * * * *