CBCS SCHEME

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17MAT41

Fourth Semester B.E. Degree Examination, Dec.2023/Jan.2024

Engineering Mathematics - IV

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find by Taylor's series method, the value of y at x = 0.1 and x = 0.2. Correct to four decimal places from $\frac{dy}{dx} = x^2y - 1$, y(0) = 1. **(06 Marks)**

b. Using modified Euler's method, solve $\frac{dy}{dx} = x + y$, y(0) = 1 at x = 0.1 for three iterations taking h = 0.1. **(07 Marks)**

c. Given $\frac{dy}{dx} = x - y^2$ and y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762, evaluate y(0.8) by Mine's method. **(07 Marks)**

OR

2 a. Use Taylor's series method to find y(0.1) from $y' = 2y + 3e^x$, y(0) = 0, considering upto fourth derivative term. **(06 Marks)**

b. Use Runge Kutta fourth order method to find y(0.2) from $10\frac{dy}{dx} = x^2 + y^2$, y(0) = 1 taking h = 0.1. **(07 Marks)**

c. Use Adam-Bashforth method to find y(1.4) from $\frac{dy}{dx} = x^2(1+y)$, y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.1979. Use corrector formulae twice. **(07 Marks)**

Module-2

3 a. If α and β are the roots of $J_n(x) = 0$ then prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$. **(06 Marks)**

b. Express $x^3 + 2x^2 - 4x + 5$ interms of Legendre polynomials. **(07 Marks)**

c. Find y(0.1) by using Runge Kutta fourth order method given that $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$, y(0) = 1, y′(0) = 0. **(07 Marks)**

OR

4 a. Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$. **(06 Marks)**

b. Find y(0.8) by using Mine's method given that $\frac{d^2y}{dx^2}=1-2y\frac{dy}{dx}$ and the following table of initial values:

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y′	0	0.1996	0.3937	0.5689

Apply corrector formula twice in presenting the value of y at x = 0.8. **(07 Marks)**

c. State and prove Rodrigue's formula. **(07 Marks)**

Module-3

5 a. Derive Cauchy-Riemann equations in Cartesian coordinates. **(06 Marks)**

b. Show that $u=\frac{1}{2}\log(x^2+y^2)$ is harmonic. Determine its harmonic conjugate. **(07 Marks)**

c. Evaluate $\int_C \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)^2(z-2)}dz$, c is $|z|=3$ by residue theorem. **(07 Marks)**

OR

6 a. If f(z)=u + iv is analytic, prove that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2+\left[\frac{\partial}{\partial y}H|(z)|\right]^2=|f'(z)|^2$. **(06 Marks)**

b. Discuss the transformation $w=e^z$. **(07 Marks)**

c. Find the bilinear transformation which sends points $z = 0, 1, \infty$ into the points $w = -5, -1, 3$ respectively. What are invariant points in this transformation? **(07 Marks)**

Module-4

7 a. The probability distribution of a finite random variable x is given by the following table.

x	−2	−1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	K

Find the value of K, mean, variance also (i) $P(x<1)$ (ii) $P(-1<x\le 2)$. **(06 Marks)**

b. Assume that on average one telephone number out of 15 called between 2 pm and 3 pm on week days is busy. What is the probability that if 6 randomly selected telephone number are called: (i) note more than three (ii) at least three of them will busy. **(07 Marks)**

c. The joint distribution of two random variable x and y as follows:

x \ y	−4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Determine: (i) The marginal distribution of x and y
(ii) Covariance of x and y
(iii) Correlation of x and y **(07 Marks)**

OR

8 a. Derive mean and variance of the exponential distribution. **(06 Marks)**

b. The life of an electric bulb is normally distributed with average life of 2000 hours and standard deviation of 60 hours out of 2500 bulbs. Find the number of bulbs that are likely to last between 1900 and 2100 hours. [Given $P(0<z<1.67)=0.4525$] **(07 Marks)**

c. The joint probability distribution of two random variables x and y are given below:

x \ y	−3	2	4
1	0.1	0.2	0.2
2	0.3	0.1	0.1

Determine: (i) E(x), E(y) and E(xy)
(ii) Covariance of x and y
(iii) Correlation of x and y **(07 Marks)**

Module-5

9 a. A die was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times on the assumption of random throwing, do the data indicate that the die is unbiased? **(06 Marks)**

b. A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure 5, 2, 8, −1, 3, 0, 6, −2, 1, 5, 0, 4 can it be calculated that the stimulus will increase the blood pressure ($t_{0.05}$ for 11 df 2.201) **(07 Marks)**

c. Find the fixed probability vector for the stochastic matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$ **(07 Marks)**

OR

10 a. Define the following terms:
(i) Null hypothesis
(ii) Type I error and Type II error
(iii) Stochastic matrix **(06 Marks)**

b. Five dice were thrown 96 times and the numbers 1, 2 or 3 appearing on the face of the dice follows the frequency distribution as below:

Number of dice showing 1, 2 or 3	5	4	3	2	1	0
Frequency	7	19	35	24	8	3

Test the hypothesis that the data follows a binomial distribution ($\chi^2_{0.05} = 11.07$ for 5 df) **(07 Marks)**

c. Three boys A, B, C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C. C is just as likely to throw the ball to B as to A. IF C was the first person to throw the ball find the probabilities that after three throws:
(i) A has the ball
(ii) B has the ball
(iii) C has the ball **(07 Marks)**

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