



CBCS SCHEME

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BMATEC301/BEC/BBM301

Third Semester B.E./B.Tech. Degree Examination, Dec.2023/Jan.2024 AV Mathematics – III for EC/BM Engineering

Time: 3 hrs.

Max. Marks: 100

- Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.
3. Statistical table and handbook permitted.
4. Use of VTU Mathematics handbook is permitted.*

		Module – 1	M	L	C															
Q.1	a.	Obtain the Fourier series of $f(x) = \frac{\pi-x}{2}$ in $0 < x < 2\pi$. Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$.	6	L2	CO1															
	b.	Expand $f(x) = 2x-1$ as a Cosine half range Fourier series in $0 < x < 1$.	7	L2	CO1															
	c.	Compute the First harmonics of the Fourier series of $f(x)$. Given the table <table border="1" style="margin: 5px auto; border-collapse: collapse;"> <tr> <td style="padding: 2px;">x</td> <td style="padding: 2px;">0</td> <td style="padding: 2px;">$\frac{\pi}{3}$</td> <td style="padding: 2px;">$\frac{2\pi}{3}$</td> <td style="padding: 2px;">π</td> <td style="padding: 2px;">$\frac{4\pi}{3}$</td> <td style="padding: 2px;">$\frac{5\pi}{3}$</td> <td style="padding: 2px;">2π</td> </tr> <tr> <td style="padding: 2px;">f(x)</td> <td style="padding: 2px;">1.0</td> <td style="padding: 2px;">1.4</td> <td style="padding: 2px;">1.9</td> <td style="padding: 2px;">1.7</td> <td style="padding: 2px;">1.5</td> <td style="padding: 2px;">1.2</td> <td style="padding: 2px;">1.0</td> </tr> </table>	x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π	f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0	7	L3
x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π													
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0													
OR																				
Q.2	a.	Obtain the Fourier series of $f(x) = x $ in $(-\ell, \ell)$.	6	L2	CO1															
	b.	Obtain the Cosine half range Fourier series of $f(x) = x^2$ in $0 < x < \pi$.	7	L3	CO1															
	c.	Express Y as a Fourier Cosine series upto second harmonics. Given the table : <table border="1" style="margin: 5px auto; border-collapse: collapse;"> <tr> <td style="padding: 2px;">x</td> <td style="padding: 2px;">0</td> <td style="padding: 2px;">2</td> <td style="padding: 2px;">4</td> <td style="padding: 2px;">6</td> <td style="padding: 2px;">8</td> <td style="padding: 2px;">10</td> <td style="padding: 2px;">12</td> </tr> <tr> <td style="padding: 2px;">y</td> <td style="padding: 2px;">9.0</td> <td style="padding: 2px;">18.2</td> <td style="padding: 2px;">24.4</td> <td style="padding: 2px;">27.8</td> <td style="padding: 2px;">27.5</td> <td style="padding: 2px;">22.0</td> <td style="padding: 2px;">9.0</td> </tr> </table>	x	0	2	4	6	8	10	12	y	9.0	18.2	24.4	27.8	27.5	22.0	9.0	7	L3
x	0	2	4	6	8	10	12													
y	9.0	18.2	24.4	27.8	27.5	22.0	9.0													
Module – 2																				
Q.3	a.	Find the Fourier transform of $f(x) = e^{- x }$.	6	L2	CO2															
	b.	Find the Fourier Cosine and Sine transform of $f(x) = e^{-\alpha x}$, $\alpha > 0$.	7	L3	CO2															
	c.	i) Find a Discrete Fourier transform of the single $f = [3, 4, 5, 5]^T$. ii) Find the Inverse Discrete Fourier transform of the single obtained in part (i).	7	L3	CO2															
OR																				
Q.4	a.	Find the Fourier transform of $f(x) = \begin{cases} 1- x & \text{for } x \leq 1 \\ 0 & \text{for } x > 1 \end{cases}$ and hence deduce that $\int_0^{\infty} \frac{\sin^2 t}{t^2} dt = \frac{\pi}{2}$.	6	L2	CO2															

	b.	Obtain the Fourier Cosine transform of $f(x) = \begin{cases} 4x & , 0 < x < 1 \\ 4-x & , 1 < x < 4 \\ 0 & , x > 4 \end{cases}$	7	L3	CO2
	c.	Solve the Integral equation $\int_0^\infty f(\theta) \cos \alpha \theta d\theta = \begin{cases} 1-\alpha & , 0 \leq \alpha \leq 1 \\ 0 & , \alpha > 1 \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2} dt$.	7	L3	CO2
Module – 3					
Q.5	a.	Find the Z – transform of i) $\cos n\theta$ ii) $\sin n\theta$.	6	L2	CO3
	b.	Find the Inverse Z – transform of $\frac{z^2 - 8z}{(z-4)^2}$.	7	L3	CO3
	c.	Solve the difference equation $y_{n+2} - 4y_n = 0$. Given that $y_0 = 0$ and $y_1 = 2$.	7	L3	CO3
OR					
Q.6	a.	Find the Z – transform of $2n + \sin\left(\frac{n\pi}{4}\right) + 1$.	6	L2	CO3
	b.	Compute the Inverse Z – transform of $\frac{3z^2 + 2z}{(5z-1)(5z+2)}$.	7	L3	CO3
	c.	Solve the difference equation $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$ with $u_0 = u_1 = 0$, using Z – transforms.	7	L3	CO3
Module – 4					
Q.7	a.	Solve $(D^4 - m^4)y = 0$.	6	L2	CO4
	b.	Solve $(D^2 - 2D + 1)y = \sin x + e^x$.	7	L3	CO4
	c.	Solve $x \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = \frac{1}{x}$.	7	L3	CO4
OR					
Q.8	a.	Solve $\frac{d^3y}{dx^3} + 8y = x^4 + 2x + 1$.	6	L2	CO4
	b.	Solve the Legendre's form of Linear equation. $(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = \sin 2 [\log(1+x)]$.	7	L3	CO4
	c.	In the LCR circuit the charge q on a plate of condenser is given by $L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = E \sin p t$. Solve the above equation.	7	L3	CO4

Module – 5																																						
Q.9	a.	Find a Least square straight line for the following data : <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>y</td> <td>6</td> <td>4</td> <td>3</td> <td>5</td> <td>4</td> <td>2</td> </tr> </table>	x	1	2	3	4	5	6	y	6	4	3	5	4	2	6	L2	CO5																			
x	1	2	3	4	5	6																																
y	6	4	3	5	4	2																																
	b.	In a partially destroyed laboratory record, the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$ and $20x - 9y = 107$. Calculate \bar{x} , \bar{y} and coefficient of correlation between x and y.	7	L3	CO5																																	
	c.	Ten competition in a beauty contest are ranked by two judges A and B in the following order. Calculate the rank correlation coefficient. <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>ID No. of competition</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> <td>10</td> </tr> <tr> <td>Judge A</td> <td>1</td> <td>6</td> <td>5</td> <td>10</td> <td>3</td> <td>2</td> <td>4</td> <td>9</td> <td>7</td> <td>8</td> </tr> <tr> <td>Judge B</td> <td>6</td> <td>4</td> <td>9</td> <td>8</td> <td>1</td> <td>2</td> <td>3</td> <td>10</td> <td>5</td> <td>7</td> </tr> </table>	ID No. of competition	1	2	3	4	5	6	7	8	9	10	Judge A	1	6	5	10	3	2	4	9	7	8	Judge B	6	4	9	8	1	2	3	10	5	7	7	L3	CO5
ID No. of competition	1	2	3	4	5	6	7	8	9	10																												
Judge A	1	6	5	10	3	2	4	9	7	8																												
Judge B	6	4	9	8	1	2	3	10	5	7																												
OR																																						
Q.10	a.	Fit a parabola for the data in the form $y = ax^2 + bx + c$. <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>y</td> <td>10</td> <td>12</td> <td>13</td> <td>16</td> <td>19</td> </tr> </table>	x	1	2	3	4	5	y	10	12	13	16	19	6	L2	CO5																					
x	1	2	3	4	5																																	
y	10	12	13	16	19																																	
	b.	The following table gives the heights of Father (x) and Sons (y) : <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>65</td> <td>66</td> <td>67</td> <td>67</td> <td>68</td> <td>69</td> <td>70</td> <td>72</td> </tr> <tr> <td>y</td> <td>67</td> <td>68</td> <td>65</td> <td>68</td> <td>72</td> <td>72</td> <td>69</td> <td>71</td> </tr> </table> <p>Find the lines of regression and hence calculate the co-efficient of correlation.</p>	x	65	66	67	67	68	69	70	72	y	67	68	65	68	72	72	69	71	7	L3	CO5															
x	65	66	67	67	68	69	70	72																														
y	67	68	65	68	72	72	69	71																														
	c.	Determine the rank correlation for the following data which shows the marks obtained in two quizzes in mathematics. <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>Marks in first quiz X</td> <td>6</td> <td>5</td> <td>8</td> <td>8</td> <td>7</td> <td>6</td> <td>10</td> <td>4</td> <td>9</td> <td>7</td> </tr> <tr> <td>Marks in first quiz Y</td> <td>8</td> <td>7</td> <td>7</td> <td>10</td> <td>5</td> <td>8</td> <td>10</td> <td>6</td> <td>8</td> <td>6</td> </tr> </table>	Marks in first quiz X	6	5	8	8	7	6	10	4	9	7	Marks in first quiz Y	8	7	7	10	5	8	10	6	8	6	7	L3	CO5											
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Marks in first quiz Y	8	7	7	10	5	8	10	6	8	6																												
