CBCS SCHEME

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15MAT31

Third Semester B.E. Degree Examination, Dec.2023/Jan.2024
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Obtain the Fourier expansion of the function $f(x) = x$ over the interval $(-\pi, \pi)$. Deduce that $\frac{\pi}{4} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)}$. **(05 Marks)**

b. Find the Fourier series expansion for the function $f(x) = (\pi - x)^2$ in the interval $(0, 2\pi)$. **(05 Marks)**

c. Compute the constant term and the first harmonic in the Fourier series of f(x) given by the following table:

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(06 Marks)

OR

2 a. Expand $f(x) = e^{-ax}$ as a Fourier series in the interval $(-\pi, \pi)$. **(05 Marks)**

b. Find the half range Fourier sine series of

$$f(x) = \begin{cases} x & \text{in} \quad 0 < x < 1 \\ 2-x & \text{in} \quad 1 < x < 2 \end{cases}$$ **(05 Marks)**

c. The following table gives the variation of a periodic current A over a period T.

t (sec)	0	T/6	T/3	T/2	2T/3	5T/6	T
A (amp)	1.98	1.3	1.05	1.3	–0.88	–0.25	1.98

Show that there is a constant part of 0.75 amp in the current A, and obtain the amplitude of the first harmonic. **(06 Marks)**

Module-2

3 a. Find the Fourier transform of the function

$$f(x) = \begin{cases} 1-|x| & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$ **(05 Marks)**

b. Find the z-transform of $\left[3n - 4\sin\left(\frac{n\pi}{4}\right)\right]$. **(05 Marks)**

c. Find inverse z-transform of $\frac{20z^3 + 3z}{(5z-1)(5z+2)}$. **(06 Marks)**

OR

4 a. Find the Fourier cosine transform of e^{-ax}. **(05 Marks)**

b. Find the Fourier sine transform of the function

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ a - x & \text{for } 1 < x < a \\ 0 & \text{for } x > a \end{cases}$$ **(05 Marks)**

c. Solve the difference equation $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0 = u_1$ by z-transform method. **(06 Marks)**

Module-3

5 a. Find the coefficient of correlation for the following data:

x	1	2	3	4	5
y	2	5	3	8	7

(05 Marks)

b. By the method of least squares, find the straight line that best fits to the following data in the form y = ax + b.

x	1	2	3	4	5
y	14	27	40	55	68

(05 Marks)

c. Find the real root of the equation $xe^x = \cos x$ that lies between 0.4 and 0.6, correct to 4 decimal places by Regula Falsi Method. **(06 Marks)**

OR

6 a. The equations of regression lines of two variables x and y are given by y = 0.516x + 33.73 and x = 0.512y + 32.52, find $\bar{x}$, $\bar{y}$ and coefficient of correlation. **(05 Marks)**

b. Fit a curve of the form $y = ae^{bx}$ for the following data:

x	5	15	20	30	35	40
y	10	14	25	40	50	62

(05 Marks)

c. Using Newton-Raphson method, find the real root of the equation $e^x = 3x$ correct to 3 decimal places, taking initial approximate root $x_0 = 0.5$. **(06 Marks)**

Module-4

7 a. Using Newton's backward interpolation formula, find the value of y(85) for the following data:

x	40	50	60	70	80	90
y	184	204	226	250	276	304

(05 Marks)

b. For the following data, find x as a polynomial in y using the inverse Lagrange's interpolation formula:

x	2	10	17
y	1	3	4

Also, find x, given the value of y = 5. **(05 Marks)**

c. Evaluate $\int_0^1 \frac{x}{1+x^2}dx$ by Simpson's 1/3rd rule by taking six equal parts. **(06 Marks)**

OR

8 a. Given f(40) = 184, f(50) = 204, f(60) = 226, f(70) = 250, f(80) = 276, find f(38) using Newton's forward interpolation formula. **(05 Marks)**

b. If f(1) = 4, f(3) = 32, f(4) = 55, f(6) = 119, find the interpolating polynomial by Newton's divided difference formula. **(05 Marks)**

c. A curve is drawn to passes through the points given by the following table:

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Using Weddle's rule, estimate the area bounded by the curve x-axis and the lines x = 1 and x = 4. **(06 Marks)**

Module-5

9 a. Using Green's theorem, evaluate $\int_C (2x^2 - y^2)dx + (x^2 + y^2)dy$, where 'c' is the triangle formed by the lines x = 0, y = 0 and x + y = 1. **(05 Marks)**

b. Using Stoke's theorem evaluate $\int_C \vec{f}.d\vec{r}$, where

$\vec{f} = (y + z - 2x)i + (z + x - 2y)j + (x + y - 2z)k$

and 'c' is the triangle with vertices (1, 0, 0), (0, 2, 0) and (0, 0, 3). **(05 Marks)**

c. State and prove Euler's equation. **(06 Marks)**

OR

10 a. Evaluate $\iint_S \vec{F}.\hat{n}ds$ where $\hat{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and 's' is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. **(05 Marks)**

b. Find the curves on which the functional $\int_0^1 [(y')^2 + 12xy]dx$, with y(0) = 0 and y(1) = −1 can be extremised. **(05 Marks)**

c. Show that the geodesics in a plane is straight line. **(06 Marks)**

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