CBCS SCHEME

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18MAT21

Second Semester B.E. Degree Examination, Dec.2023/Jan.2024
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find grade 'ϕ' when ϕ is given by $\phi = 3x^2y - y^3z^2$ at the point (1, -2, -1). **(06 Marks)**

b. A vector field is given by $\vec{A} = (x^2 + xy^2)\ \hat{i} + (y^2 + x^2y)j$. Show that the field is irrotational. **(07 Marks)**

c. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2, -1, 2). **(07 Marks)**

OR

2 a. Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$, where C is the closed curve bounded by $y = x$ and $y = x^2$. **(06 Marks)**

b. Evaluate by Stokes theorem $\oint_C yzd_x + zxd_y + xyd_z$, where C is the curve $x^2 + y^2 = 1$, $z = y^2$. **(07 Marks)**

c. Using the divergence theorem, evaluate $\iint_S \vec{F}.\hat{n}\,ds$, where $\vec{F} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. **(07 Marks)**

Module-2

3 a. Solve $\frac{d^3y}{dx^3} + y = 0$. **(06 Marks)**

b. Solve $y'' - 4y' + 13y = \cos 2x$. **(07 Marks)**

c. Solve $\frac{d^2y}{dx^2} + y = \tan x$ by the method of variation or parameters. **(07 Marks)**

OR

4 a. Solve $x^2y'' - xy' - xy' + 2y = x$ by Cauchy method. **(06 Marks)**

b. Solve $(2x + 1)^2\ y'' - 2(2x + 1)y' - 12y = 6x$ by Lagendre's method. **(07 Marks)**

c. A particle moves along the x – axis according to the law $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 25x = 0$. If the particle is started at x = 0 with an initial velocity of 12 ft/sec to the left, determine nets. **(07 Marks)**

Module-3

5 a. Form partial differential equation by eliminating the arbitrary constants 'a' & 'b'. $z = ax^2 + by^2$. **(06 Marks)**

b. Form partial differential equation by eliminating the arbitrary function 'f'. $z = x^n.f\left(\frac{y}{x}\right)$ **(07 Marks)**

c. Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \sin(2x + 3y)$. **(07 Marks)**

OR

6 a. Solve $\frac{\partial^3 z}{\partial x^2} + 4z = 0$. Given that when x = 0 , $z = e^{2y}$ and $\partial z/\partial x = 2$. **(06 Marks)**

b. Solve p cotx + q coty = cotz. **(07 Marks)**

c. Find solution of one – dimensional heat equation :

$\frac{\partial u}{\partial t} = c^2 . \frac{\partial^2 y}{\partial x^2}$. **(07 Marks)**

Module-4

7 a. Discuss the convergence of $\sum_{n=1}^{\infty} \frac{(n+1)^n x^n}{n^{n+1}}$. **(06 Marks)**

b. Test for convergence of the series $\frac{1^2}{2} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + \frac{4^2}{2^4} + \ldots\ldots$ **(07 Marks)**

c. Test the positive series +=1 + 2 + 3 + ……+ n. **(07 Marks)**

OR

8 a. Solve Bessel's differential equation leading to $J_n(x)$. **(06 Marks)**

b. Express the polynomial $f(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre polynomials. **(07 Marks)**

c. Using Rodrigues's formula, obtain expressions for $P_0(x)$, $P_1(x)$, $P_2(x)$, $P_3(x)$. **(07 Marks)**

Module-5

9 a. Using Newton's forward interpolation formula, find y at x = 8 from the following table :

x :	0	5	10	15	20	25
y :	7	11	14	18	24	32

(06 Marks)

b. Using Newton's divided difference formula, evaluate f(8) and f(15), given

x :	4	5	7	10	11	13
f(x) :	48	100	294	900	1210	2028

(07 Marks)

c. Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ by Regula Falsi method correct to three decimal places. **(07 Marks)**

OR

10 a. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Weddle's Rule. **(06 Marks)**

b. Evaluate $\int_2^8 \log_{10}^x dx$ taking 6 subintervals correct to four decimal places by Simpson's $\left(3/8\right)^{th}$ rule. **(07 Marks)**

c. Use Newton – Raphson method to find a real root of the equation $x e^x - 2 = 0$ correct to three decimal places. **(07 Marks)**

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