CBCS SCHEME

USN

18MAT11

First Semester B.E. Degree Examination, Dec.2023/Jan.2024
Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Show that the angle ϕ between radius vector and tangent at a point on the curve $r = f(\theta)$ is given by $\tan\phi = r.\frac{d\theta}{dr}$. **(06 Marks)**

b. Show that the radius of curvature of the curve $x^3 + y^3 = 3xy$ at $\left(\frac{3}{2}, \frac{3}{2}\right)$ is $-\frac{3}{8\sqrt{2}}$. **(06 Marks)**

c. Find the angle of intersection between the curves $r = a\log\theta$ and $r = \frac{a}{\log\theta}$. **(08 Marks)**

OR

2 a. Find the pedal equation of the curve $r^m = a^m[\cos m\theta + \sin m\theta]$. **(06 Marks)**

b. Find the radius of curvature of the curve $r^n = a^n \sin n\theta$. **(06 Marks)**

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x-2a)^3$. **(08 Marks)**

Module-2

3 a. Using Maclaurin's series, prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + ...$ **(08 Marks)**

b. Evaluate $\lim_{x\to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{\frac{1}{x}}$. **(06 Marks)**

c. Examine the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$ for its extreme values. **(06 Marks)**

OR

4 a. If $U = f(x-y, y-z, z-x)$, prove that $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} = 0$. **(06 Marks)**

b. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. **(07 Marks)**

c. A rectangular box, open at the top, is to have a volume of 32 cubic ft. Find the dimension of the box requiring least material for its construction. **(07 Marks)**

Module-3

5 a. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$. **(06 Marks)**

b. Evaluate $\iint xy(x+y)\,dy\,dx$, taken over the area between $y = x^2$ and $y = x$. **(07 Marks)**

c. Show that $\int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta = \pi$. **(07 Marks)**

OR

6 a. Change the order of integration in $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dxdy$, and hence evaluate the same. **(06 Marks)**

b. Find by double integration, the centre of gravity of the area of the cardioid $r = a(1 + \cos\theta)$. **(07 Marks)**

c. Derive the relation between Beta and Gamma functions as $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

Module-4

7 a. Solve $\dfrac{dy}{dx} + y\tan x = y^2 \sec x$. **(06 Marks)**

b. Find the orthogonal trajectories of the family $r^n \cos n\theta = a^n$. **(07 Marks)**

c. Solve the equation $(px - y)(py + x) = 2p$ by reducing into Clairaut's form, taking the substitution $X = x^2$, $Y = y^2$. **(07 Marks)**

OR

8 a. If the temperature of the air is 30°C and a metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach a temperature of 40°C. **(06 Marks)**

b. Find the orthogonal trajectories of the family of curves $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2+\lambda} = 1$, where λ is the parameter. **(07 Marks)**

c. Solve $xy\left(\dfrac{dy}{dx}\right)^2 - (x^2 + y^2)\dfrac{dy}{dx} + xy = 0$. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ by applying elementary row operations. **(06 Marks)**

b. Reduce the matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ into the diagonal form. **(07 Marks)**

c. Find the Largest eigen value and the corresponding eigen vector of the matrix A, by using the power method by taking initial vector as $[1, 1, 1]^T$,

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Perform 6 iterations. **(07 Marks)**

OR

10 a. Apply Gauss-Jordan method to solve the following system of equations:

$2x + y + 3z = 1$

$4x + 4y + 7z = 1$

$2x + 5y + 9z = 3$ **(06 Marks)**

b. Investigate for what value of λ and μ the simultaneous equation $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have :

(i) no solutions

(ii) unique solutions

(iii) infinite number of solutions **(07 Marks)**

c. Solve the following system of equations by Gauss-Seidel method

$5x + 2y + z = 12$

$x + 4y + 2z = 15$

$x + 2y + 5z = 20$

Carryout 4 iterations taking the initial approximation to the solution as (1, 0, 3). **(07 Marks)**

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