CBCS SCHEME

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17MAT11

First Semester B.E. Degree Examination, Dec.2023/Jan.2024
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the n^{th} derivative of $\dfrac{6x}{(x-2)(x+2)(x-1)}$. **(06 Marks)**

b. Prove that the curves $r = a(1+\cos\theta)$, $r = b(1-\cos\theta)$ cuts orthogonally. **(07 Marks)**

c. Find the radius of curvature for the curve $y^2 = \dfrac{4a^2(2a-x)}{x}$, where the curve cuts the x-axis. **(07 Marks)**

OR

2 a. If $y = a\cos(\log x) + b\sin(\log x)$ then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(06 Marks)**

b. In usual notation, prove that $\tan\phi = r\dfrac{d\theta}{dr}$. **(07 Marks)**

c. Find the Pedal equation of the curve $r^n = a^n \cos n\theta$. **(07 Marks)**

Module-2

3 a. Obtain the Taylor's series of $\log_e x$ in powers of $(x-1)$ upto fourth degree. **(06 Marks)**

b. State Euler's theorem, use the same prove that $xu_x+yu_y=2u\log u$ where $\log u = \dfrac{x^3+y^3}{3x+4y}$. **(07 Marks)**

c. If $u = x+3y^2-z^2$, $v = 4x^2yz$, $w = 2z^2-xy$. Evaluate $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, -1, 0). **(07 Marks)**

OR

4 a. Evaluate $\underset{x\to 0}{\mathrm{Lt}}\left(\dfrac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$. **(06 Marks)**

b. Find the Maclaurians expansion of log(secx) upto fourth degree term. **(07 Marks)**

c. If $u = f\left(\dfrac{x}{y},\dfrac{y}{z},\dfrac{z}{x}\right)$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}+z\dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

Module-3

5 a. A particle moves along the curve $x = 2t^2$, $y = t^2-4t$, $z = 3t-5$. Find the velocity and acceleration at t = 1. Also find component of velocity and component of acceleration in the direction of $\hat{i}+3\hat{j}+2\hat{k}$. **(06 Marks)**

b. Find constants 'a' and 'b' such that $\vec{F} = (axy+z^3)i + (3x^2-z)j + (bxz^2-y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Find the angle between the tangents to the curve $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3k$ at the point $t = \pm 1$. **(07 Marks)**

OR

6 a. Find $\text{div}\,\vec{f}$ and $\text{curl}\,\vec{f}$ where, $\vec{f} = \text{grad}(x^3+y^3+z^3-3xyz)$. **(06 Marks)**

b. If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, prove that $\nabla(r^n) = nr^{n-2}\,\vec{r}$. **(07 Marks)**

c. Prove that $\nabla.\left(\phi\vec{A}\right) = \phi\left(\nabla.\vec{A}\right) + \nabla\phi.\vec{A}$. **(07 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \sin^n x\,dx$, hence evaluate $\int_0^{\frac{\pi}{2}} \sin^n x dx$. **(06 Marks)**

b. Solve $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$. **(07 Marks)**

c. If the temperature of the air is 30°C and the substance cools from 100°C to 70°C in 15 minutes, find when the temperature will be 40°C. **(07 Marks)**

OR

8 a. Evaluate $\int_0^{\frac{\pi}{6}} \cos^4 3x \sin^2 6x\,dx$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$. **(07 Marks)**

c. Find the orthogonal trajectory of $r^n = a^n \cos n\theta$. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix,

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

Using elementary row operations. **(06 Marks)**

b. Solve $2x+y+4z=12$, $4x+11y-z=33$, $8x-3y+2z=20$ by Gauss elimination method. **(07 Marks)**

c. Find the largest eigen value and the corresponding eigen vector for $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ taking the initial vector as $[1 \quad 1 \quad 1]^T$ carry out 5 iterations by using power method. **(07 Marks)**

OR

10 a. Solve $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$ by Gauss-Seidel method. Carry out 3 iterations. **(06 Marks)**

b. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form. **(07 Marks)**

c. Reduce the quadratic form, $3x^2 + 5y^2 + 3z^2 - 2xy + 2zx - 2yz$ to the canonical form, using orthogonal transformation. **(07 Marks)**

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