**CBCS SCHEME**

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**BMATM101**

# First Semester B.E./B.Tech. Degree Examination, Dec.2023/Jan.2024
## Mathematics – I for ME Stream

Time: 3 hrs. Max. Marks: 100

*Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.*
*2. VTU Formula Hand Book is permitted.*
*3. M : Marks , L: Bloom's level , C: Course outcomes.*

| | | Module – 1 | M | L | C |
|---|---|---|---|---|---|
| Q.1 | a. | Prove that with usual notations $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}(dr/d\theta)^2$ | 06 | L1 | CO1 |
| | b. | Find the pedal equation $r^m = a^m \cos m\theta$ | 07 | L2 | CO1 |
| | c. | Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1, 1). | 07 | L3 | CO1 |
| | | **OR** | | | |
| Q.2 | a. | Derive the radius of curvature in Cartesian form as $\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$ | 08 | L1 | CO1 |
| | b. | Show that for the curve $r = a(1 - \cos\theta)$ is $\frac{\rho^2}{r} =$ constant. | 08 | L3 | CO1 |
| | c. | Using modern mathematical tool write a program/code to plot the sine and cosine curve. | 04 | L3 | CO5 |
| | | **Module – 2** | | | |
| Q.3 | a. | Expand $e^x$ by Maclaurin's series upto the term containing $x^4$. | 06 | L2 | CO2 |
| | b. | If $u = f(x - y, y - z, z - x)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ | 07 | L2 | CO2 |
| | c. | Find the extreme values of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ | 07 | L3 | CO2 |
| | | **OR** | | | |
| Q.4 | a. | Evaluate $\lim_{x \to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$ | 08 | L2 | CO2 |
| | b. | If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ | 08 | L3 | CO2 |
| | c. | Using modern mathematical tool write a program/code to evaluate $\lim_{x \to 0}\left(1 + 1/x\right)^x$ | 04 | L3 | CO5 |
| | | **Module – 3** | | | |
| Q.5 | a. | Solve $\frac{dy}{dx} - y\tan x = y^2 \sec x$ | 06 | L2 | CO3 |
| | b. | Solve $(x^2 + y^2 + x)dx + xy\,dy = 0$ | 07 | L3 | CO3 |
| | c. | Water at temperature 10°C takes 5 minutes to warm up to 20°C at room temperature of 40°. Find the temperature of the water after 20 minutes. | 07 | L2 | CO3 |

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|---|---|---|---|---|---|
| | | **OR** | | | |
| **Q.6** | **a.** | Solve $\frac{dy}{dx}-\frac{dx}{dy}=\frac{x}{y}-\frac{y}{x}$ | **06** | **L2** | **CO3** |
| | **b.** | Find the orthogonal trajectories of cardioid $r = a(1-\cos\theta)$ | **07** | **L2** | **CO3** |
| | **c.** | Find the general solution of $xp^2 + xp - yp + 1 = 0$ | **07** | **L3** | **CO3** |
| | | **Module – 4** | | | |
| **Q.7** | **a.** | Solve $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$ | **06** | **L2** | **CO4** |
| | **b.** | Solve $(D^2 - 4)y = \cos 2x + e^{3x}$ | **07** | **L2** | **CO4** |
| | **c.** | Solve $(D^2 + 1)y = \sec x$ by the method of variation of parameter. | **07** | **L3** | **CO4** |
| | | **OR** | | | |
| **Q.8** | **a.** | Solve $(D^3 + 1)y = 3 + 5e^x$ | **06** | **L2** | **CO4** |
| | **b.** | Solve $(D^2 + D)y = x^2 + 2x + 4$ | **07** | **L2** | **CO4** |
| | **c.** | Solve $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=\log x$ | **07** | **L3** | **CO4** |
| | | **Module – 5** | | | |
| **Q.9** | **a.** | Find the rank of $\begin{bmatrix}1 & 2 & 4 & 3\\ 2 & 4 & 6 & 8\\ 4 & 8 & 12 & 16\\ 1 & 2 & 3 & 4\end{bmatrix}$ | **06** | **L2** | **CO5** |
| | **b.** | Using Gauss - Jordan method, solve $x + 2y + z = 8$, $2x + 3y + 4z = 20$, $4x + 3y + 2z = 16$ | **07** | **L2** | **CO5** |
| | **c.** | Find the largest eigen value and the corresponding eigen vector of $\begin{bmatrix}2 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 2\end{bmatrix}$ with initial approximate eigen vector $(1, 0, 0)^T$. Carry out 4 iterations. | **07** | **L3** | **CO5** |
| | | **OR** | | | |
| **Q.10** | **a.** | Investigate for what values of $\lambda$ and $\mu$, so that the equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ gave (i) no solution (ii) a unique solution and (iii) an infinite nnumber of solutions. | **07** | **L2** | **CO5** |
| | **b.** | Solve the system of equations using Gauss-Siedel method by taking $(0, 0, 0)$ as an initial approximate root. $5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$. Carry out 4 iterations. | **08** | **L3** | **CO5** |
| | **c.** | Using modern mathematical tool write a program/code to find the largest eigen value of $A = \begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$ by power method. | **05** | **L3** | **CO5** |

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