CBCS SCHEME

USN 1AY15IS042

15CS36

Third Semester B.E. Degree Examination, Dec.2023/Jan.2024

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Prove that for any propositions p, q, r the compound proposition : $\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$ is a tautology. **(06 Marks)**

b. Prove the following logical equivalence using the laws of logic: $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg (q \vee p)$. **(05 Marks)**

c. Prove the following logical equivalence using the laws of logic: $[\neg p \wedge (\neg q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$. **(05 Marks)**

OR

2 a. Prove the validity of the arguments using rule of inference :

$$\begin{array}{l} (\neg p \vee \neg q) \to (r \wedge s) \\ r \to t \\ \neg t \\ \hline \therefore p \end{array}$$

(05 Marks)

b. Test the validity of the arguments using rule of inference.

$$\begin{array}{l} (\neg p \vee q) \to r \\ r \to (s \vee t) \\ \neg s \wedge \neg u \\ \neg u \to \neg t \\ \hline \therefore p \end{array}$$

(05 Marks)

c. Find whether the following argument is valid :

No Engineering student of 1st or 2nd semester studies logic
Anil is an Engineering student who studies logic

∴ Anil is not in second semester. **(06 Marks)**

Module-2

3 a. Prove by mathematical induction, for every positive integer 8 divides $5^n + 2 \cdot 3^{n-1} + 1$. **(06 Marks)**

b. Assuming PASCAL language is case insensitive, an identifier consists of a single letter followed by upto seven symbols which may be letters or digits (26 letters, 10 digits). There are 36 reserved words. How many distinct identifiers are possible in this version of PASCAL? **(05 Marks)**

c. Find the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$. **(05 Marks)**

OR

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

4 a. Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. **(05 Marks)**

b. Lucas numbers are defined recursively as $L_0 = 2$, $L_1 = 1$ and $L_n = L_{n-1} + L_{n-2}$ for $n \geq 2$. If Fi^s are fibonacci numbers and Li^s are the Lucas numbers, prove that $L_n = F_{n-1} + F_{n+1}$ for all positive integers n. **(05 Marks)**

c. Find the number of distinct terms in the expansion of $(w + x + y + z)^{12}$. **(06 Marks)**

Module-3

5 a. Let $f : R \rightarrow R$ be defined by

$$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \leq 0 \end{cases}$$

determine $f(0)$, $f(-1)$, $f^{-1}(0)$, $f^{-1}(+3)$, $f^{-1}([-5, 5])$. **(08 Marks)**

b. Define an Equivalence Relation. Write the partial order relation for the positive divisors of 36 and write its Hasse diagram (HASSE). **(08 Marks)**

OR

6 a. Consider the function $f : R \rightarrow R$ defined by $f(x) = 2x + 5$. Let a function $g : R \rightarrow R$ be defined by $g(x) = \frac{1}{2}(x - 5)$. Prove that g is an inverse of f. **(03 Marks)**

b. State Pigeonhole principle. Let ABC is an equilateral triangle whose sides are of length 1cm each. If we select 5 points inside the triangle, prove that atleast two of their points are such that the distance between them is less than ½ cm. **(05 Marks)**

c. If A = {1, 2, 3, 4}, R and S are relations on A defined by R = {(1, 2), (1, 3), (2, 4), (4, 4)} S = {(1, 1), (1, 2), (1,3), (1, 4), (2, 3), (2, 4)} find RoS, SoR, R^2, S^2 and write down their matrices. **(08 Marks)**

Module-4

7 a. In a survey of 260 college students, the following data were obtained. 64 had taken mathematics course, 94 had taken CS course, 58 had taken EC course, 28 had taken both Mathematics and EC course, 26 had taken both Mathematics and CS course, 22 had taken both CS and EC course, and 14 had taken all three types of course. Determine how many of these students had taken none of the three subjects. **(05 Marks)**

b. Find the rook polynomial for the 3 × 3 board using expansion formula. **(06 Marks)**

c. Solve the recurrence relation :

$a_n + a_{n-1} - 6a_{n-2} = 0 \quad n \geq 2$, given $a_0 = -1$ and $a_1 = 8$. **(05 Marks)**

OR

8 a. An apple, a banana, a mango and an orange are to be distributed among 4 boys B_1, B_2, B_3, B_4. The boys B_1 and B_2 do not wish to have an apple, the boy B_3 does not want banana or mango and B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased. **(06 Marks)**

b. How many permutation of 1, 2, 3, 4, 5, 6, 7, 8 are not derangements? **(04 Marks)**

c. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(06 Marks)**

Module-5

9 a. Define the following with an example :

i) Simple graph ii) Regular graph iii) Subgraph

iv) Maximal subgraph v) Induced subgraph. **(05 Marks)**

b. Show that there exists no simple graphs corresponding to the following degree sequences :
i) 0, 2, 2, 3, 4
ii) 1, 1, 2, 3
iii) 2, 3, 3, 4, 5, 6
iv) 2, 2, 4, 6. **(04 Marks)**

c. Let T = (V, E) be a complete m-ary tree with |V| = n. If T has ℓ leaves and i internal vertices, then prove the following :
i) $n = m \cdot i + 1$
ii) $\ell = (m-1)i + 1$
iii) $i = \dfrac{(\ell - 1)}{(m-1)} = \dfrac{(n-1)}{m}$. **(07 Marks)**

OR

10 a. In the graph shown in Fig. Q10(a). Determine
i) a walk from b to d that is not a trail
ii) b – d trail that is not a path
iii) a path from b to d
iv) a closed walk from b to b that is not a circuit
v) a circuit from b to b that is not a cycle
vi) a cycle from b to b. **(06 Marks)**

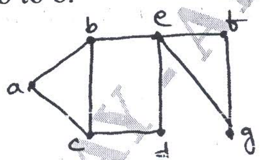

Fig. Q10(a)

b. Determine the order |V| of the graph G = (V, E) in the following cases :
i) G is cubic graph with 9 edges
ii) G is regular with 15 edges
iii) G has 10 edges with 2 vertices of degree 4 and all other of degree 3. **(06 Marks)**

c. Obtain the optimal prefix code for the string ROAD IS GOOD. **(04 Marks)**

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