18MATDIP31

USN

Additional Mathematics - I

Time: 3 hrs.   Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1   a.   Express the complex number $\dfrac{(3+i)(1-3i)}{2+i}$ in the form x + iy. Also find its magnitude.
                                                                                (06 Marks)
    b.   Find the cube roots of ℓ - i and represent them in an argand plane.   **(07 Marks)**
    c.   If $\vec{a} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{b} = 8\hat{i} - 4\hat{j} + \hat{k}$ then show that $\vec{a}$ is perpendicular to $\vec{b}$, also find $|\vec{a} \times \vec{b}|$.
                                                                                (07 Marks)

OR

2   a.   Find the modulus and amplitude of $1 - \cos\alpha + i\sin\alpha$.   **(06 Marks)**
    b.   If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$; $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} - \hat{k}$, find
         i) $\vec{a} \cdot (\vec{b} \times \vec{c})$ ii) $\vec{b} \times (\vec{a} \times \vec{c})$.   **(07 Marks)**
    c.   Prove that $[\vec{a} \times \vec{b}, \ \vec{b} \times \vec{c}, \ \vec{c} \times \vec{a}] = [\vec{a} \ \vec{b} \ \vec{c}]^2$.   **(07 Marks)**

Module-2

3   a.   Using Maclaurin's series, prove that $\sqrt{1 + \sin 2x} = 1 + x - \dfrac{x^2}{2} - \dfrac{x^3}{6} + \dfrac{x^4}{24} - - - -$.   **(06 Marks)**

    b.   If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x - y}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$.   **(07 Marks)**

    c.   If $u = 1 - x$, $v = x(1-y)$, $w = xy(1-z)$, find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$.   **(07 Marks)**

OR

4   a.   Obtain the Maclaurin's expansion of the function $\log(1 + e^x)$.   **(06 Marks)**
    b.   If $u = f(x-y, y-z, z-x)$, Prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$.   **(07 Marks)**

    c.   If $u = x + y + z$, $w = y + z$, $z = uvw$, find $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$.   **(07 Marks)**

Module-3

5   a.   A particle moves along a curve C with parametric equations $x = t - \dfrac{t^3}{3}$, $y = t^2$ and $z = t + \dfrac{t^3}{3}$, where t is the time. Find the velocity and acceleration and any time t and also find their magnitudes at $t = 3$.   **(06 Marks)**
    b.   Find div $\vec{F}$ and Curl $\vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$.   **(07 Marks)**
    c.   Find the directional derivative of $\phi = x^2 yz^3$ at (1, 1, 1) in the direction of $\hat{i} + \hat{j} + 2\hat{k}$.
                                                                                (07 Marks)

OR

6 a. Show that the vector field $\vec{F} = yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k}$ is solenoidal vector field. (06 Marks)

b. If $\vec{F} = (x + y + 1)\,\hat{i} + \hat{j} - (x + y)\,\hat{k}$, show that $\vec{F} \cdot \text{curl } \vec{F} = 0$. (07 Marks)

c. Find the constants a , b , c such that $\vec{F} = (x + y + az)\,\hat{i} + (x + cy + 2z)\,\hat{k} + (bx + 2y - z)\,\hat{j}$ is irrotational. (07 Marks)

Module-4

7 a. Obtain the Reduction formula for $\int_0^{\pi/2} \cos^n x \, dx$. (06 Marks)

b. Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)\, dy\, dx$. (07 Marks)

c. Evaluate $\int_0^1 \int_0^1 \int_0^1 (x + y + z)\, dx\, dy\, dz$. (07 Marks)

OR

8 a. Evaluate $\int_1^2 \int_0^{3-y} xy \, dx \, dy$. (06 Marks)

b. Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} \, dx\, dy\, dz$. (07 Marks)

c. Obtain the Reduction formula $\int \sin^m x \cos^n x \, dx$. (07 Marks)

Module-5

9 a. Solve : $(x^2 + y)\, dx + (y^3 + x)\, dy = 0$. (06 Marks)

b. Solve : $x \log x \dfrac{dy}{dx} + y = 2 \log x$. (07 Marks)

c. Solve : $\dfrac{dy}{dx} + \dfrac{y}{x} = y^2 x$. (07 Marks)

OR

10 a. Solve : $y\, e^y\, dx = (y^3 + 2x\, e^y)\, dy$. (06 Marks)

b. Solve : $(x^2 - y^2)\, dx = 2xy\, dy$. (07 Marks)

c. Solve : $[1 + (x + y) \tan y] \dfrac{dy}{dx} + 1 = 0$. (07 Marks)

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