CBCS SCHEME

USN

17MATDIP31

Third Semester B.E. Degree Examination, June/July 2023
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Prove that $\left[\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right]^4 = \cos 8\theta + i\sin 8\theta$. **(06 Marks)**

b. Express $\frac{1}{(2+i)^2} - \frac{1}{(2-i)^2}$ in the form of $a + ib$. **(06 Marks)**

c. If $\vec{a} = 4i + 3j + k$, $\vec{b} = 2i - j + 2k$. Find a unit vector N perpendicular to vectors $\vec{a}$ and $\vec{b}$ such that $\vec{a}$, $\vec{b}$, $\vec{N}$ form a right handed system. Also find the angle between the vectors $\vec{a}$ and $\vec{b}$. **(08 Marks)**

OR

2 a. Show that $(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$. **(06 Marks)**

b. Find the value of λ so that the vectors $\vec{a} = i - 2j + 3k$, $\vec{b} = 7i + j + k$ and $\vec{c} = 3i + 4j - k$ are coplanar. **(06 Marks)**

c. Define scalar and vector products of two vectors. If $\vec{a} = 2i - 3j - k$ and $\vec{b} = i + 4j - 2k$. Find $\vec{a}.\vec{b}$ and $\vec{a} \times \vec{b}$. **(08 Marks)**

Module-2

3 a. If $y = \tan^{-1} x$, prove that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. **(06 Marks)**

b. If $u = x^2y + y^2z + z^2x$, prove that $u_x + u_y + u_z = (x+y+z)^2$. **(06 Marks)**

c. Show that the pair of curves $r = a(1+\cos\theta)$, $r = b(1-\cos\theta)$ intersect orthogonally. **(08 Marks)**

OR

4 a. Find the pedal equation to the curve $r^m = a^m \cos m\theta$. **(06 Marks)**

b. If $u = e^{\left(\frac{x^2y^2}{x+y}\right)}$, prove that $xu_x + yu_y = 3u\log u$. **(06 Marks)**

c. Obtain the MaClaurin's series expansion of $y = \sin x + \cos x$ upto the term containing x^4. **(08 Marks)**

Module-3

5 a. Obtain a reduction formula for $\int \sin^n x dx$, $n > 0$. **(06 Marks)**

b. Evaluate $\int_0^a \frac{x^4}{\sqrt{a^2-x^2}}dx$. (06 Marks)

c. Evaluate $\int_0^1\int_0^{1-x}\int_0^{1-x-y} xy\,dz\,dy\,dx$. (08 Marks)

OR

6 a. Evaluate $\int_0^1 \frac{x^9}{\sqrt{1-x^2}}dx$. (06 Marks)

b. Evaluate $\int_0^1\int_1^{x^2}\left(x^3+y^2x\right)dx\,dy$. (06 Marks)

c. Evaluate $\int_{-1}^1\int_0^z\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$. (08 Marks)

Module-4

7 a. If $\vec{F}=(x+y+1)i+j-(x+y)k$, show that $\vec{F}.\text{curl}\,\vec{F}=0$. (06 Marks)

b. Find the directional derivative of $\phi=x^2yz+4xz^2$ at the point $(1,-2,-1)$ in the direction of the vector $2i-j-2k$. (06 Marks)

c. A particle moves along a curve $x=e^{-t}$, y = 2cos3t, z = 2sin3t, where t is the time variable. Determine its velocity and acceleration and also magnitudes of velocity and acceleration at t = 0. (08 Marks)

OR

8 a. Find the values of the constants a, b, c such that $\vec{F}=(x+y+az)i+(bx+2y-z)j+(x+cy+2z)k$ is irrational. (06 Marks)

b. Show that the vector field $\vec{F}=\frac{xi+yj}{x^2+y^2}$ is solenoidal. (06 Marks)

c. Find div $\vec{F}$ and curl $\vec{F}$ where $\vec{F}=\nabla(x^3+y^3+z^3-3xyz)$. (08 Marks)

Module-5

9 a. Solve $\left(x^2+y\right)dx+\left(y^2+x\right)dy=0$. (06 Marks)

b. Solve $\frac{dy}{dx}+y\tan x=\sec x$. (06 Marks)

c. Solve $x^2ydx-\left(x^3+y^3\right)dy=0$. (08 Marks)

OR

10 a. Solve $\frac{dy}{dx}+\frac{y}{x}=y^2x$. (06 Marks)

b. Solve $\frac{dy}{dx}=\frac{y}{x}+\cos^2\left(\frac{y}{x}\right)$. (06 Marks)

c. Solve $3x\left(x+y^2\right)dy+\left(x^3-3xy-2y^3\right)dx=0$. (08 Marks)

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