CBCS SCHEME

USN 1 A Y 2 1 M T 4 1 9

18MAT41

Fourth Semester B.E. Degree Examination, June/July 2023
Complex Analysis, Probability and Statistical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find analytic function u + iv, where u is given to be $u = e^x[(x^2 - y^2)\cos y - 2xy \sin y]$. **(06 Marks)**

b. Derive Cauchy Reimann equations in polar form. **(07 Marks)**

c. Show that $u = e^{2x}[x\cos 2y - y\sin 2y]$ is harmonic. Find the analytic function f(z) = u + iv. **(07 Marks)**

OR

2 a. Derive Cauchy Reimann equation in Cartesian form. **(06 Marks)**

b. Determine analytic function f(z) = u + iv if $u - v = e^x[\cos y - \sin y]$. **(07 Marks)**

c. Show that $w = z^n$ is analytic and hence find its derivative. **(07 Marks)**

Module-2

3 a. Discuss the transformation $w = z + \frac{1}{z}, z \neq 0$. **(06 Marks)**

b. Find the Bilinear transformation which maps the points z = 1, i, -1 onto w = 0, 1, ∞. **(07 Marks)**

c. Evaluate $\int_0^{2+i} (\bar{z})^2 \, dz$ along i) line y = x/2 ii) real axis to 2 and then vertically to 2 + iy. **(07 Marks)**

OR

4 a. Discuss the transformation $w = z^2$. **(06 Marks)**

b. State and prove Cauchy's integral formula $f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{(z-a)} dz$. **(07 Marks)**

c. Evaluate using Cauchy's integral formula.

$$\int_C \frac{e^{2z}}{(z-1)(z-2)} dz \quad C: |z| = 3.$$ **(07 Marks)**

Module-3

5 a. Define: i) Random variable ii) Discrete probability distribution with an example. **(06 Marks)**

b. The probability that man aged 60 will live upto 70 is 0.65. What is the probability that out of 10 men, now aged 60 i) Exactly 9 ii) atmost 9 iii) Atleast 7 will live up to age of 70 years. **(07 Marks)**

c. In a normal distribution, 3% of items are under 45 and 8% are over 64. Find the mean and standard deviation, given that A(0.5) = 0.19 and A(1.4) = 0.42. **(07 Marks)**

OR

6 a. The probability distribution of a finite random variable X is given by

X :	-2	-1	0	1	2	3
P(x) :	0.1	K	0.2	2K	0.3	K

Find 'K', mean and variance of X. **(06 Marks)**

b. If probability of bad reaction from certain injection is 0.001. Determine the chance that out of 2000 individuals more than two will get bad reaction, and less than two will get bad reaction. **(07 Marks)**

c. The frequency of accidents per shift in a factory is shown in the following table:

Accidents per shift	0	1	2	3	4
Frequency	192	100	24	3	1

Calculate mean numbers of accidents per shift. Find the corresponding Poisson distribution. **(07 Marks)**

Module-4

7 a. Fit a second degree parabola $y = a + bx + cx^2$ for the following data:

x	0	1	2	3	4	5
y	1	3	7	3	21	31

(06 Marks)

b. Find the coefficient of correlation, lines of regression of x on y and y on x. Given,

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(07 Marks)

c. If θ is an acute angle between line of regression, then show that $\tan\theta = \frac{\sigma x}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)$.

Indicate the significance of the cases r = 0 and r = ±1. **(07 Marks)**

OR

8 a. Fit the curve of the form ax^b and hence estimate y when x = 8.

x	5	10	15	20	25	30	35
y	2.76	3.17	3.44	3.64	3.81	3.95	4.07

(06 Marks)

b. Find the rank correlation coefficient for the following data:

x	93	44	53	08	71	81	6	10	32	31
y	45	62	12	28	92	84	73	3	51	32

(07 Marks)

c. With the usual notations compute $\bar{x}$, $\bar{y}$ and r from the following lines of regression: $y = 0.516x + 33.73$ and $x = 0.512y + 32.52$. **(07 Marks)**

Module-5

9 a. The joint probability distribution for following data

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine the marginal distributions of X and Y also calculate E(x), E(y), COV (xy). **(06 Marks)**

b. Define: i) Null hypothesis ii) Confidence limits iii) Type I, Type II errors. **(07 Marks)**

c. The following table gives the distribution of digits in the numbers chosen at random from a telephone directory:

Digits	0	1	2	3	4	5	6	7	8	9
Frequency	1026	1107	997	966	1075	933	1107	972	964	853

Test whether the digits may be taken to occur equally frequently in the directory. (given $\chi^2_{0.05} = 16.92$ at n = 9). **(07 Marks)**

OR

10 a. A fair coin is tossed thrice. The random variable X and Y are defined as follows. X = 0 or 1 according as head or tail occurs on first loss, Y = number of heads.
i) Determine distribution of X and Y.
ii) Joint probability distribution of X and Y.
iii) Expectation of X, Y and XY. **(06 Marks)**

b. It is claimed that a random sample of 49 tyres has a mean life of 15200km. Is the sample drawn from population whose mean is 15,150km and standard deviation is 200km? Test the significance level at 0.05 level. **(07 Marks)**

c. Ten individuals are choosen at random from the population and their height in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of universe is 66′ (value of $t_{0.05} = 2.262$ for 9.D.F). **(07 Marks)**

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