



# CBCS SCHEME

18MAT31

## Third Semester B.E. Degree Examination, June/July 2023 Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

### Module-1

- 1 a. Find  $L\left(\frac{\cos at - \cos bt}{t}\right)$ . (06 Marks)
- b. Express the function in terms of unit step function and hence find Laplace transform of
- $$f(t) = \begin{cases} \sin t & 0 < t < \frac{\pi}{2} \\ \cos t & \frac{\pi}{2} < t < \pi \end{cases}$$
- (07 Marks)
- c. Solve  $y''(t) + 4y'(t) + 3y(t) = e^t$ ,  $y(0) = y'(0) = 1$  by using Laplace transform method. (07 Marks)

OR

- 2 a. Find : (i)  $L^{-1}\left(\log\left(\frac{s+b}{s+a}\right)\right)$  (ii)  $L^{-1}\left(\frac{s+3}{s^2-4s+13}\right)$  (06 Marks)
- b. Find  $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$  by using convolution theorem. (07 Marks)
- c. Given  $f(t) = \begin{cases} t & 0 < t < a \\ 2a-t & a < t < 2a \end{cases}$
- where  $f(t) = f(t+2a)$  then show that  $L(f(t)) = \frac{1}{s^2} \tan h\left(\frac{as}{2}\right)$  (07 Marks)

### Module-2

- 3 a. Obtain Fourier series for  $f(x) = \frac{\pi-x}{2}$ ,  $0 < x < 2\pi$ . (06 Marks)
- b. Find Fourier series for  $f(x) = 2x - x^2$ ,  $0 < x < 2$ . (07 Marks)
- c. Find half range Fourier cosine series for
- $$f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi-x, & \frac{\pi}{2} < x < \pi \end{cases}$$
- (07 Marks)

OR

- 4 a. Find Fourier series for  $f(x) = |x|$ ,  $-\pi < x < \pi$ . (06 Marks)
- b. Obtain Fourier series for  $f(x) = \begin{cases} 0 & -2 < x < 0 \\ 1 & 0 < x < 2 \end{cases}$ . (07 Marks)
- c. Find the Fourier series upto first harmonic from the following table:

x	0	1	2	3	4	5
y = f(x)	4	8	15	7	6	2

(07 Marks)

**Module-3**

- 5 a. Find Fourier transform of
- $f(x)$
- , given:

$$f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases} \text{ and hence deduce that } \int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}.$$

(06 Marks)

- b. Find the Fourier cosine transform of

$$f(x) = \begin{cases} 4x & 0 < x < 1 \\ 4-x & 1 < x < 4 \\ 0 & x > 4 \end{cases}$$

(07 Marks)

- c. Solve
- $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$
- , given
- $u_0 = 0, u_1 = 1$
- using Z - transform.

(07 Marks)

OR

- 6 a. Find the Fourier sine transform of
- $e^{-|x|}$
- and hence evaluate
- $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx$
- .

(06 Marks)

- b. Find Z-transform of
- $\cos n\theta$
- and
- $a^n \cos n\theta$
- .

(07 Marks)

- c. Obtain the inverse Z-transform of
- $\frac{2z^2 + 3z}{(z+2)(z-4)}$
- .

(07 Marks)

**Module-4**

- 7 a. Find the value of
- $y$
- at
- $x = 0.1$
- and
- $x = 0.2$
- given
- $\frac{dy}{dx} = x^2y - 1, y(0) = 1$
- by using Taylor's series method.

(06 Marks)

- b. Compute
- $y(0.1)$
- , given
- $\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1$
- taking
- $h = 0.1$
- , by using Runge-Kutta 4
- <sup>th</sup>
- order method.

(07 Marks)

- c. Find the value of
- $y$
- at
- $x = 0.4$
- , given
- $\frac{dy}{dx} = 2e^x - y$
- with initial conditions
- $y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.04, y(0.3) = 2.09$
- by using Milne's predictor and corrector method.

(07 Marks)

OR

- 8 a. Using modified Euler's method, find the value of
- $y$
- at
- $x = 0.1$
- , given
- $\frac{dy}{dx} = -xy^2, y(0) = 2$
- taking
- $h = 0.1$
- .

(06 Marks)

- b. Solve
- $\frac{dy}{dx} = 3e^x + 2y, y(0) = 0$
- at
- $x = 0.1$
- taking
- $h = 0.1$
- , by using Runge-Kutta 4
- <sup>th</sup>
- order method.

(07 Marks)

- c. Find the value
- $y$
- at
- $x = 0.8$
- given
- $\frac{dy}{dx} = x - y^2$
- and

x	0	0.2	0.4	0.6
y	0	0.0200	0.0795	0.1762

By using Adam's Bashforth predictor and corrector method.

(07 Marks)

**Module-5**

- 9 a. Solve  $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$  for  $x = 0.2$  given  $x = 0, y = 1$  and  $\frac{dy}{dx} = 0$  by using Runge-Kutta method. (07 Marks)
- b. Derive Euler's equation in the standard form  $\frac{\partial f}{\partial y} = \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$ . (06 Marks)
- c. Find the extremal of the function  $\int_0^1 [(y')^2 + 12xy] dx$  with  $y(0) = 0$  and  $y(1) = 1$ . (07 Marks)

OR

- 10 a. Find the value of  $y$  at  $x = 0.8$ , given  $\frac{d^2y}{dx^2} = 2y \frac{dy}{dx}$  and

x	0	0.2	0.4	0.6
y	1	0.2027	0.4228	0.6841
y'	1	1.041	1.179	1.468

- by using Milne's method. (07 Marks)
- b. Prove that the shortest between two points in a plane is a straight line. (06 Marks)
- c. Find the curve on which the functional  $\int_0^1 [x + y + (y')^2] dx$  with  $y(0) = 1, y(1) = 2$ . (07 Marks)

\*\*\*\*\*