

# CBCS SCHEME

BMATM201



## Second Semester B.E./B.Tech. Degree Examination, June/July 2023 Mathematics-II for ME Stream

Time: 3 hrs.

Max. Marks: 100

- Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.  
2. VTU Formula Hand Book is permitted.  
3. M : Marks , L: Bloom's level , C: Course outcomes.*

Module - 1			M	L	C
Q.1	a.	Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dy dx$ .	7	L3	CO1
	b.	Evaluate $\int_0^a \int_x^{\sqrt{a^2-y^2}} y\sqrt{x^2+y^2} dx dy$ by changing into polar.	7	L3	CO1
	c.	Show that $\gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ .	6	L2	CO1
<b>OR</b>					
Q.2	a.	Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy dy dx$ by changing the order of integration.	7	L3	CO1
	b.	Using double integration find the area of the plane in the form of a quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .	7	L3	CO1
	c.	Using modern mathematical tools, write a program to evaluate : $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx$	6	L3	CO5
<b>Module - 2</b>					
Q.3	a.	If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ , find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ .	7	L2	CO2
	b.	Find the angle between the surfaces $x^2 + y^2 - z^2 = 4$ and $z = x^2 + y^2 - 13$ at $(2, 1, 2)$ .	7	L2	CO2
	c.	Define a irrotational vector. Find the constants a, b, c such that $\vec{F} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$ is irrotational.	6	L2	CO2
<b>OR</b>					
Q.4	a.	If $\vec{F} = xy\hat{i} + yz\hat{j} + zx\hat{k}$ , evaluate $\int_C \vec{F} \cdot d\vec{r}$ where c is the curve represented by $x = t, y = t^2, z = t^3, -1 \leq t \leq 1$ .	7	L3	CO2
	b.	Using Green's theorem, evaluate $\oint (3x^2 - 8y^2)dx + (4y - 6xy)dy$ , where c is the boundary of the region enclosed by $y = \sqrt{x}$ and $y = x^2$ .	7	L3	CO2
	c.	Write the Modern mathematical tool program to find the divergence of the vector field $\vec{F} = x^2yz\hat{i} + y^2zx\hat{j} + z^2xy\hat{k}$ .	6	L3	CO5

Module – 3																	
Q.5	a.	Form the PDE by eliminating the arbitrary function from $f(x + y + z, x^2 + y^2 + z^2) = 0$	7	L2	CO3												
	b.	Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2 \sin y$ , when $x = 0$ and $z = 0$ if $y$ is an odd multiple of $\frac{\pi}{2}$ .	7	L3	CO13												
	c.	Derive one dimensional wave equation.	6	L2	CO3												
OR																	
Q.6	a.	Form the PDE by eliminating the arbitrary constants $a$ and $b$ from $(x - a)^2 + (y - b)^2 + z^2 = r^2$ .	7	L2	CO3												
	b.	Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that $x = 0, z = 0$ and $\frac{\partial z}{\partial x} = a \sin y$ .	7	L3	CO3												
	c.	Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$ , using Lagrange's multipliers.	6	L3	CO3												
Module – 4																	
Q.7	a.	Compute the real root of the equation $x \log_{10} x - 1.2 = 0$ by the Regula Falsi method taking four decimal places. Perform three approximation.	7	L3	CO4												
	b.	The population of a town is given by the following table: <table border="1" style="margin-left: 20px;"> <tr> <td>Year</td> <td>1951</td> <td>1961</td> <td>1971</td> <td>1981</td> <td>1991</td> </tr> <tr> <td>Population</td> <td>19.96</td> <td>39.65</td> <td>58.81</td> <td>72.21</td> <td>94.61</td> </tr> </table> Using Forward and Backward Necoton's interpolation formula, calculate the increase in population between the years 1955 to 1985.	Year	1951	1961	1971	1981	1991	Population	19.96	39.65	58.81	72.21	94.61	7	L3	CO4
Year	1951	1961	1971	1981	1991												
Population	19.96	39.65	58.81	72.21	94.61												
	c.	Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ by taking 7 ordinates using Trapezoidal rule.	6	L3	CO4												
OR																	
Q.8	a.	Using the Newton Raphson method, find the real root of the equation $x \sin x + \cos x = 0$ , which is nearer to $x = \pi$ , correct to three decimal places.	7	L3	CO4												
	b.	Compute the value of $y$ when $x = 4$ using Lagrange's interpolation formula given <table border="1" style="margin-left: 20px;"> <tr> <td><math>x</math></td> <td>0</td> <td>2</td> <td>3</td> <td>6</td> </tr> <tr> <td><math>f(x)</math></td> <td>-4</td> <td>2</td> <td>14</td> <td>158</td> </tr> </table>	$x$	0	2	3	6	$f(x)$	-4	2	14	158	7	L3	CO4		
$x$	0	2	3	6													
$f(x)$	-4	2	14	158													
	c.	Evaluate $\int_0^{\pi/2} \sqrt{\cos \theta} d\theta$ by taking 7 ordinates using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule.	6	L3	CO4												
Module – 5																	
Q.9	a.	Use Taylor's series method to find $y(0.1)$ from $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$ .	7	L3	CO4												
	b.	Using Runge-Kutta method of order 4, find $y$ at $x = 0.2$ by taking $h = 0.2$ and given $\frac{dy}{dx} = 3x + \frac{y}{2}$ , $y(0) = 1$ .	7	L3	CO4												

	<b>c.</b>	Applying Milne's predictor and corrector method, find $y(0.8)$ from $\frac{dy}{dx} = x - y^2$ and given $y(0) = 0$ , $y(0.2) = 0.02$ , $y(0.4) = 0.0795$ , $y(0.6) = 0.1762$ .	6	L3	CO4
<b>OR</b>					
<b>Q.10</b>	<b>a.</b>	Solve by using modified Euler's method $\frac{dy}{dx} = x - y^2$ , $y(0) = 1$ taking $h = 0.1$ , find $y(0.2)$ .	7	L3	CO4
	<b>b.</b>	Using Runge-Kutta method of order 4, find $y$ at $x = 0.1$ , given that $\frac{dy}{dx} = 3e^x + 2y$ , $y(0) = 0$ and $h = 0.1$ .	7	L3	CO4
	<b>c.</b>	Using mathematical tools, write a code to find the solution of $\frac{dy}{dx} = 1 + \frac{y}{x}$ at $y(2)$ taking $h = 0.2$ given that $y(1) = 2$ by Runge-Kutta method of order 4.	6	L3	CO4

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