CBCS SCHEME

USN | | | | | | | | | | | 18MAT11

First Semester B.E. Degree Examination, June/July 2023
Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. With usual notations, prove that $\frac{1}{p^2}=\frac{1}{r^2}+\frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$. (06 Marks)

b. Find the radius of curvature of the curve $y^2=\frac{a^2(a-x)}{x}$ at the point (a, 0). (06 Marks)

c. For the curve $\theta=\frac{1}{a}\sqrt{r^2-a^2}-\cos^{-1}\left(\frac{a}{r}\right)$, prove that $p^2=r^2-a^2$. (08 Marks)

OR

2 a. Find the angle between the curves $r=a(1-\cos\theta)$ and $r=2a\cos\theta$. (06 Marks)

b. Find the radius of curvature of the curve $r=a\sin n\theta$ at the pole (0, 0). (06 Marks)

c. Find evolutes curve $y^2=4ax$ as $27ay^2=4(x+a)^3$. (08 Marks)

Module-2

3 a. Obtain Maclaurin's expansion of $e^{\tan^{-1}x}$ upto the term containing x^4. (06 Marks)

b. Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$. (06 Marks)

c. Find the extreme values of $f(x, y)=x^3y^2(1-x-y)$. (08 Marks)

OR

4 a. If U = f(x-y, y-z, z-x), prove that $\frac{\partial U}{\partial x}+\frac{\partial U}{\partial y}+\frac{\partial U}{\partial z}=0$. (06 Marks)

b. If $u=x+3y^2-z^2$, $v=x^2yz$, $w=2z^2-xy$, find the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, -1, 0). (06 Marks)

c. Find the stationary values of $x^2+y^2+z^2$ subject to the condition $xy+yz+zx=3a^2$. (08 Marks)

Module-3

5 a. Evaluate $\int_0^1\int_{y^2}^1\int_0^{1-x} x\,dz\,dx\,dy$. (07 Marks)

b. Find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, above x-axis. (07 Marks)

c. With usual notations, prove that $\beta(m,n)=\frac{\Gamma m\Gamma n}{\Gamma(m+n)}$. (06 Marks)

OR

6 a. Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dx\,dy$ by changing the order of integration. (07 Marks)

b. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$ by changing into polar coordinates. (07 Marks)

c. Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\,d\theta \times \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$. (06 Marks)

Module-4

7 a. Solve $p(p+y) = x(x+y)$. (07 Marks)

b. Find the orthogonal trajectories to the family of curve, $y^2 = 4ax$. (07 Marks)

c. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$. (06 Marks)

OR

8 a. Solve $(x^2+y^2+x)dx + xydy = 0$. (07 Marks)

b. A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C. What will be the temperature of the body after 40 minutes from the original? (07 Marks)

c. Find the general solution and singular solution of the equation $\sin px \cos y = \cos px \sin y + p$. (06 Marks)

Module-5

9 a. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ by reducing to row-reduced echelon form. (06 Marks)

b. Apply Gauss-elimination method to solve the $x+4y-z=-5$, $x+y-6z=-12$, $3x-y-z=4$. (07 Marks)

c. Find numerically largest eigen value and corresponding eigen vector of $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by Rayleigh's power method. Take initial eigen vector $[1,0,0]^T$. Carry out five iterations. (07 Marks)

OR

10 a. Test for consistency and solve the system of equations, $x+y+z=6$, $x-y+2z=5$, $3x+y+z=8$. (06 Marks)

b. Solve the system of equations by Gauss-Seidel method $x+y+54z=110$, $27x+6y-z=85$, $6x+15y+2z=72$. Carryout three iterations. (07 Marks)

c. Diagonalize the matrix $\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. (07 Marks)

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