CBCS SCHEME

USN

18EC44

Fourth Semester B.E. Degree Examination, June/July 2023
Engineering Statistics and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Discuss the CDF and PDF of a random variable. List the properties of PDF. **(08 Marks)**

b. Given the data in the following table:

k	1	2	3	4	5
y_k	2.1	3.2	4.8	5.4	6.9
$P\{y_k\}$	0.2	0.21	0.19	0.14	0.26

i) Plot the PDF and CDF of the discrete random variable Y.

ii) Write expressions for PDF and CDF using unit delta and unit-step functions. **(08 Marks)**

c. A continuous random variable X has a PDF, $f_x(x) = 3x^2 \quad 0 \le x \le 1$. Find 'a' such that $P\{x > a\} = 0.05$. **(04 Marks)**

OR

2 a. Define an exponential random variable. Obtain the characteristic function of an exponential random variable and using the characteristic function derive its mean and variance. **(10 Marks)**

b. Given the data in the following table:

k	1	2	3	4	5
y_k	2.1	3.2	4.8	5.4	6.9
$P(y_k)$	0.2	0.21	0.19	0.14	0.26

i) What are the mean and variance of Y.

ii) If $W = y^2 + 1$, what are mean and variance of W. **(10 Marks)**

Module-2

3 a. Define correlation coefficient of random variables x and y. Show that it is bounded by limits ± 1. **(05 Marks)**

b. The joint PDF $f_{xy}(x, y) = C$, a constant when $0 < x < 3$ and $0 < y < 3$ and is '0' otherwise.

i) What is the value of the constant 'C'?

ii) What are the PDFs for X and Y?

iii) What $F_{xy}(x, y)$ when $0 < x < 3$ and $0 < y < 3$?

iv) What are $F_{xy}(x, \infty)$ and $F_{xy}(\infty, y)$?

v) Are x and y independent? **(10 Marks)**

c. Prove that COV (ax, by) = ab cov(xy). **(05 Marks)**

OR

4 a. Define central limit theorem and show that the sum of two independent Gaussian random variables is also Gaussian. **(06 Marks)**

b. For a bivariate random variable CDF is given by $c(x+1)^2(y+1)^2$ for $\begin{cases} -2 < x < 4, \\ -1 < y < 2 \end{cases}$ and "0" outside. Find:
   i) The value of 'c'
   ii) Bivariate PDF
   iii) $F_x(x)$ and $F_y(y)$
   iv) Evaluate $P\{(x \le 2) \cap (y \le 1)\}$
   v) Are there variables independent? **(10 Marks)**

c. Explain briefly the following random variables:
   i) Chi-square random variable
   ii) Student-t random variable. **(04 Marks)**

Module-3

5 a. Define random process, with help of examples discuss different types of random processes. **(08 Marks)**

b. Explain strict-sense-stationary and wide-sense-stationary random process. **(04 Marks)**

c. A random process is defined by $x(t) = A \sin(w_c t + \ominus)$ where A, w_c are constants and $\ominus$ is a uniformly distributed random variable, distributed between $-\pi$ and π. Check whether x(t) is WSS. If yes list its mean and ACF. **(08 Marks)**

OR

6 a. Define Auto Correlation Function (ACF) of a random process and discuss its properties. **(10 Marks)**

b. The random process x(t) and y(t) are jointly wide-sense stationary and independent. Given that W(t) = x(t) + y(t) and

$$R_x(\tau) = 10e^{-\frac{|\tau|}{3}}$$

$$R_y(\tau) = 10^{\left(\frac{3-|\tau|}{3}\right)} \quad -3 \le \tau \le 3$$

$= 0$ (otherwise).

For W(t), find i) ACF ii) Total power iii) ac power iv) dc power v) check whether W(t) is W.S.S. **(10 Marks)**

Module-4

7 a. Define vector space and explain four fundamental subspaces with example. **(08 Marks)**

b. Determine the column space and null space of the matrix $B = \begin{bmatrix} 0 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix}$. **(06 Marks)**

c. Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ to the Echelon (u) form and find the rank of the matrix. **(06 Marks)**

OR

8 a. What is basis for a vector space? Explain. **(06 Marks)**

b. Given the vectors (1, -3, 2), (2, 1, -3) and (-3, 2, 1). Identify the basis. Verify they are independent or not. **(08 Marks)**

c. Determine orthonormal vectors for $u = \begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix}$. **(06 Marks)**

Module-5

9 a. By applying row operations to produce upper triangular matrix u, compute $|A|$ (det A).

$$A = \begin{bmatrix} 3 & 1 & 4 & 2 \\ 1 & 5 & 2 & 6 \\ 2 & 3 & 7 & 1 \\ 4 & 1 & 2 & 3 \end{bmatrix}.$$ **(08 Marks)**

b. For the given upper triangular matrix, determine i) $|u|$ ii) $|u^T|$ iii) $|u^{-1}|$.

$$u = \begin{bmatrix} 4 & 4 & 2 & 8 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$ **(06 Marks)**

c. What is cofactor? Explain with an example. **(06 Marks)**

OR

10 a. Find x, y and z using CRAMER's rule for the system of equations,

$x + 4y - z = 1$

$x + y + z = 0$

$2x + 3z = 0.$ **(06 Marks)**

b. Determine the eigen values of matrix $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$. **(04 Marks)**

c. i) List the properties of Singular Value Decomposition (SVD).

ii) Prove that Identity matrix is positive definite using all required tests. **(10 Marks)**

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