CBCS SCHEME

USN [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

21EC33

Third Semester B.E. Degree Examination, June/July 2023
Basic Signal Processing

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Define vector space and list out the eight rules that satisfies addition and scalar multiplication. **(05 Marks)**

b. For which right hand side vector (b_1, b_2, b_3) have solution to the system.

$$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$ **(08 Marks)**

c. Define column space and null space of the matrix. **(07 Marks)**

OR

2 a. Determine the complete solution $x = x_n + x_p$ to the system

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$ **(05 Marks)**

b. Find the best straight line fit (least square) to the measurement $b = 4$ at $t = -2$, $b = 3$ at $t = -1$, $b = 1$ at $t = 0$ and $b = 0$ at $t = 2$. Then find the projection of b on to the column space of A **(08 Marks)**

c. Apply the Gram – Schmidt process for the independent vectors

$a = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $c = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ to obtain an orthonormal basis. **(07 Marks)**

Module-2

3 a. Find the eigen values and eigen vectors of $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$. Check that $\lambda_1 + \lambda_2 + \lambda_3$ equals the trace and $\lambda_1\ \lambda_2\ \lambda_3$ equals the determinent. **(08 Marks)**

b. For the matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$, solve the differential equation $\frac{du}{dt} = Au, u(0) = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$. What are the two pure exponential solutions? **(12 Marks)**

OR

4 a. If $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$ and eigen vector matrix $S = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix}$. Determine the diagonalization matrix $\wedge = S^{-1}AS$ **(08 Marks)**

b. For the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$, find the eigen values, eigen vector v_1, v_2 and $A^T A$. Then find u_1, u_2 and recover A using Singular Value Decomposition (SVD). **(12 Marks)**

Module-3

5 a. Define signals and systems. **(04 Marks)**

b. $x(n) = [\underset{\uparrow}{2}, 2, 2, 2, -2, -2, -2, -2]$. Sketch i) $x(n-3)$ ii) $x(2n+3)$. **(06 Marks)**

c. Determine whether the system $y(n) = nx(n)$ is
   i) Stable
   ii) Memory
   iii) Causal
   iv) Time invariant
   v) Linear **(10 Marks)**

OR

6 a. Sketch the signal $x(n) = u(n+10) - 2u(n) + u(n-6)$
$y(n) = 2n[u(n) - u(n) - 6)]$ **(10 Marks)**

b. Sketch the following signals
   i) $x(2n)$
   ii) $x(3n-1)$
   iii) $x(n)\,u(1-n)$ if $x(n) = [3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3]$ **(10 Marks)**

Module-4

7 a. Derive an expression for convolution sum for Linear Time Invariant (LTI) system. **(04 Marks)**

b. Compute $y(n) = u(n) * u(n)$ using graphical method. **(08 Marks)**

c. Compute $y(n) = x(n) * h(n)$, where $x(n) = u(n)$ and $h(n) = \left(\frac{3}{4}\right)^n u(n)$ using graphical method. **(08 Marks)**

OR

8 a. Show that convolution posses the associative and distributive property. **(08 Marks)**

b. For the impulse response $h(n) = 2u(n) - 2u(n-5)$. Determine whether the system
   i) Memoryless
   ii) Stable
   iii) Causal **(06 Marks)**

c. What is step response? Evaluate the step response of the LTI system whose impulse response in $h(n) = \left(\frac{1}{2}\right)^n u(n)$. **(06 Marks)**

Module-5

9 a. Find the z-transform and mention ROC of the following signals
   i) $x(n) = [\underset{\uparrow}{1}, 2, 3, 4, 0, 7]$
   ii) $x(n) = [1, 2, 3, \underset{\uparrow}{4}, 0, 7]$
   iii) $x(n) = [1, 2, 3, 4, 0, \underset{\uparrow}{7}]$ **(03 Marks)**

b. Find the z-transform of the signal $x(n) = a^n u(-n-1)$ with ROC diagram. **(05 Marks)**

c. Using the properties of the z-transform, find the z-transform of the following signals

i) $x(n) = a^n \cos \Omega_0 n\, u(n)$

ii) $x(n) = u(n-2) * \left(\frac{2}{3}\right)^n u(n)$ **(12 Marks)**

OR

10 a. Using partial fraction expansion method find the inverse z-transform of

$$x(z) = \frac{1 - z^{-1} + z^{-2}}{\left(1 - \frac{1}{2}z^{-1}\right)(1 - 2z^{-1})(1 - z^{-1})} \text{ for}$$

i) ROC $1 < |z| < 2$

ii) ROC $\frac{1}{2} < |z| < 2$

iii) ROC $|z| < \frac{1}{2}$ **(08 Marks)**

b. A causal system has an input $x(n) = \delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{\delta}\delta(n-2)$ and output $y(n) = \delta(n) - \frac{3}{4}\delta(n-1)$. Find the transfer function of the system. **(04 Marks)**

c. The LTI system is $H(z) = \frac{3 - 4z^{-1}}{1 - 3.5z^{-1} + 1.5z^{-2}}$. Specify ROC of H(z) and determine h(n) for the following conditions

i) The system is stable

ii) The system is causal

iii) The system is anticausal **(08 Marks)**

* * * * *