# CBCS SCHEME

USN

22MCA11

## First Semester MCA Degree Examination, Jan./Feb. 2023

## Mathematical Foundation for Computer Applications

Time: 3 hrs. Max. Marks: 100

*Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.*
*2. M : Marks , L: Bloom's level , C: Course outcomes.*

| | | Module – 1 | M | L | C |
|---|---|---|---|---|---|
| Q.1 | a. | Define, cardinality of a set, singleton set and universal set with example. | 6 | L2 | CO1 |
| | b. | Define union and intersection of two sets with example. | 4 | L2 | CO1 |
| | c. | Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$. | 10 | L2 | CO1 |
| | | OR | | | |
| Q.2 | a. | Define matrix. Explain different types of matrices with example. | 8 | L2 | CO1 |
| | b. | Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{6, 7, 8, 9, 10\}$ and $f : A \to B$ be a function defined by $f = \{(1, 7)(2, 7)(3, 8)(4, 6)(5, 9)(6, 9)\}$. Determine $f^{-1}(6)$ and $f^{-1}(9)$. Also if $B_1 = \{7, 8\}$, $B_2 = \{8, 9, 10\}$ then find $f^{-1}(B_1)$ and $f^{-1}(B_2)$. | 4 | L1 | CO1 |
| | c. | In a class of 52 students, 30 are studying C++, 28 are studying pascal and 13 are studying both languages. How many in this class are studying at least one of these languages? How many are studying neither of these languages? | 6 | L2 | CO1 |
| | d. | State and explain Pigeon hole principle. | 2 | L1 | CO1 |
| | | **Module – 2** | | | |
| Q.3 | a. | State the laws of logic. | 8 | L2 | CO2 |
| | b. | Write the contra positive, converse and the inverse of the conditional statement. "If oxygen is a gas then Gold is compound". | 6 | L1 | CO2 |
| | c. | Define Tautology. Show that the compound proposition, $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is a Tautology. | 6 | L3 | CO2 |
| | | OR | | | |
| Q.4 | a. | Prove the following is valid argument :<br>$p \to r$<br>$\neg p \to q$<br>$q \to s$<br>$\therefore \neg r \to s$ | 8 | L2 | CO2 |

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|---|---|---|---|---|---|
| | b. | Give the direct proof of the following statement "If n is an odd integer, then $n^2$ is odd". | 6 | L2 | CO2 |
| | c. | What is a proposition? Let p and q be the propositions "Swimming in the New Jersy sea shore is allowed and sharks have been near the sea shore." Express each of the following compound propositions as an English sentence.<br>(i) $p \to \sim q$ (ii) $\sim p \to \sim q$ (iii) $p \leftrightarrow q$ | 6 | L1 | CO2 |

**Module – 3**

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| **Q.5** | a. | Let A = {1, 2, 3, 4}, R = {(1, 3)(1, 1)(3, 1)(1, 2)(3, 3)(4, 4)} be a relation on A. Determine whether R is reflexive, symmetric, asymmetric and write matrix representation. | 6 | L2 | CO3 |
| | b. | If A = {1, 2, 3, 4} and R = {(1, 2)(1, 3)(2, 4)(4, 4)}, S = {(1, 1)(1, 2)(1, 3)(1, 4)(2, 3)(2, 4)} be relations on A then find RoS, SoR, $R^2$ and $S^2$. Also write their matrices. | 8 | L2 | CO3 |
| | c. | Discuss briefly on partitions and equivalence classes with example. | 6 | L2 | CO3 |

**OR**

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| **Q.6** | a. | Show that the set A={1, 2, 3, 4, 6, 8, 12} is a POSET with respect to the relation R defined as {(a, b) : a divides b} and draw its Hasse diagram. | 8 | L3 | CO3 |
| | b. | Draw the directed graph of relation, R = {(1, 1)(1, 3)(2, 1)(2,3)(2, 4)(3, 1)(3, 2)(4, 1)} on the set {1, 2, 3 4}. Also find in-degree and out-degree of each vertex. | 6 | L2 | CO3 |
| | c. | Define lattices. Determine whether the POSET ({1, 2, 3, 4, 5}, 1) is lattice or not. | 6 | L2 | CO3 |

**Module – 4**

<table>
<tr><td><b>Q.7</b></td><td>a.</td><td>A random variable X has the following probability distribution.
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>P(X)</td><td>K</td><td>3K</td><td>5K</td><td>7K</td><td>9K</td><td>11K</td><td>13K</td></tr>
</table>
(i) Find K.<br>
(ii) Evaluate P(X<4), $P(X \geq 5)$, $P(3 < X \leq 6)$.<br>
(iii) Find the minimum value of K so that $P(X \leq 2) > 0.3$</td><td>10</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td><b>b.</b></td><td>The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that, (i) Exactly two will be defective (ii) at least two will be defective (iii) none will be defective.</td><td>10</td><td>L2</td><td>CO4</td></tr>
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**OR**

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|---|---|---|---|---|---|
| **Q.8** | a. | Given that 2% of the fuses manufactured by a firm are defective. Find by using Poisson distribution, the probability that a box containing 200 fuses has (i) no defective fuses (ii) 3 or more defective fuses (iii) at least one defective fuse. | 8 | L2 | CO4 |

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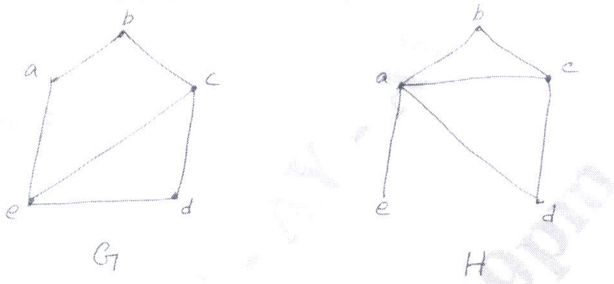
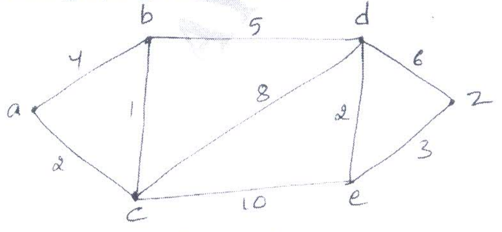
| | **b.** | The length of a telephone conversation has an exponential distribution with a mean of 3 minutes. Find the probability that a call<br>(i) Ends in less than 3 minutes.<br>(ii) Takes between 3 and 5 minutes. | 6 | L2 | CO4 |
| | **c.** | Find the constant C such that, $f(x) = \int_{-\infty}^{\infty} \begin{cases} Cx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$<br>is a probability density function. Also compute $P(1 < X < 2)$. | 6 | L2 | CO4 |
| | | **Module – 5** | | | |
| **Q.9** | **a.** | Define the following with suitable examples:<br>(i) Simple graph<br>(ii) Complete graph<br>(iii) Bipartite graph | 6 | L2 | CO5 |
| | **b.** | Verify the following graphs are isomorphic or not.<br><br> <br>Fig.Q9 (b) | 6 | L2 | CO5 |
| | **c.** | Explain the Konigsberg bridge problem. | 8 | L1 | CO5 |
| | | **OR** | | | |
| **Q.10** | **a.** | Determine $\|V\|$ for the graph G = (V, E) if G has 10 edges with two vertices of degree 4 and others of degree 3. | 4 | L2 | CO6 |
| | **b.** | Define the following with suitable examples :<br>(i) Euler's graph (ii) Hamilton graph | 6 | L2 | CO6 |
| | **c.** | Use Dijkstra's algorithm to find the length of a shortest path between the vertices a and z in the graph given below.<br><br> <br>Fig. Q10 (c) | 10 | L2 | CO6 |

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