CBCS SCHEME

USN

21MAT11

First Semester B.E. Degree Examination, Jan./Feb. 2023
Calculus and Differential Equation

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Derive with usual notations $\frac{1}{p^2}=\frac{1}{r^2}+\frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$. (06 Marks)

b. Prove that the curves $r^n = a^n \cos n\theta$ and $r^n = a^n \sin n\theta$ intersect orthogonally. (07 Marks)

c. Find the radius of curvature of the curve $x^4 + y^4 = 2$ at (1, 1). (07 Marks)

OR

2 a. Derive an expression for radius of curvature in Cartesian form. (06 Marks)

b. Find the angle of intersection between two curves $r = a\sec^2(\theta/2)$ and $r = b\,\mathrm{cosec}^2(\theta/2)$. (07 Marks)

c. Find the radius of curvature of the curve $r^2 = a^2\cos 2\theta$. (07 Marks)

Module-2

3 a. Expand $\log(1 + \sin x)$ by Maclaurin's series upto 4th degree terms. (06 Marks)

b. If $Z = xy^2 + x^2y$, where $x = at^2$, $y = 2at$, find the total derivative $\frac{dz}{dt}$. (07 Marks)

c. Find the maximum value of the function $f(x, y) = x^3y^2(1 - x - y)$ for $x \neq 0$, $y \neq 0$. (07 Marks)

OR

4 a. Expand $\log(1 + e^x)$ using Maclaurin's series upto 4th degree terms. (06 Marks)

b. Evaluate $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x^2}$. (07 Marks)

c. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$ and $w = x + y + z$, find the value of Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. (07 Marks)

Module-3

5 a. Solve $\frac{dy}{dx}+\frac{y}{x}=y^2x$. (06 Marks)

b. Find the orthogonal trajectories of the family of curves $x^2 + y^2 + 2\lambda x + C = 0$, λ - parameter. (07 Marks)

c. Solve $x^2p^2 + 3xyp + 2y^2 = 0$ where $P=\frac{dy}{dx}$. (07 Marks)

OR

6 a. Solve $y(y + x)dx + (x + 2y - 1)\,dy = 0$. **(06 Marks)**

b. A body originally at 80°C cools down to 60°C in 20 min, the temperature of the air being 40°C. What will be the temperature of the body after 40 min from the original? **(07 Marks)**

c. Solve $(y - px)(p - 1) = p$ by reducing to Clairaut's form. **(07 Marks)**

Module-4

7 a. Solve $(D^3 - 4D^2 + 4D)\,y = 0$. **(06 Marks)**

b. Solve $(D - 2)^2 y = 8(e^{2x} + 3)$. **(07 Marks)**

c. Solve $x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - 3y = x^2$. **(07 Marks)**

OR

8 a. Solve $(D^3 - 6D^2 + 11D - 6)y = e^{-2x} + e^{-3x}$. **(06 Marks)**

b. Apply the method of variation of parameters to solve $\dfrac{d^2y}{dx^2} + 4y = 4\sec 2x$. **(07 Marks)**

c. Solve $(1+x)^2\dfrac{d^2y}{dx^2} + (1+x)\dfrac{dy}{dx} + y = 2\sin\log(1+x)$. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$. **(06 Marks)**

b. Apply Gauss-Jordan method to solve the system of linear equations:

$x + y + z = 9$

$x - 2y + 3z = 8$

$2x + y - z = 3$ **(07 Marks)**

c. Use Gauss-Seidel method to solve the system of linear equations iteratively (3 iterations).

$20x + y - 2z = 17$

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$ **(07 Marks)**

OR

10 a. Test the following system for consistency and solve if the system is consistent:

$x + 2y + 3z = 1$

$2x + 3y + 8z = 2$

$x + y + z = 3$ **(06 Marks)**

b. Use Gauss elimination method to solve the system of equations

$x + 4y - z = -5,\ x + y - 6z = -12,\ 3x - y - z = 4$. **(07 Marks)**

c. Determine the largest Eigen value and the corresponding Eigen vector of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Choose initial eigen vector as $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^1$. Carryout 5 iterations. **(07 Marks)**

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