USN ☐☐☐☐☐☐☐☐☐☐ **MATDIP401**

Fourth Semester B.E. Degree Examination, April 2023
Advanced Mathematics – II

Time: 3 hrs.   Max. Marks:100

Note: *Answer any FIVE full questions.*

1 a. Find the angle between any two diagonals of a cube. **(06 Marks)**

b. Show that the points $A(-4,9,6)$, $B(-1,6,6)$ and $C(0,7,10)$ from a right angled isosceles triangle. **(06 Marks)**

c. If $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are direction cosines of a line, then prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$. **(08 Marks)**

2 a. Find the equation of the plane which passes through the point $(3, -3, 1)$ and is normal to the line joining the points $(3, 2, -1)$ and $(2, -1, 5)$. **(06 Marks)**

b. Derive the equation of the plane in the intercept form $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$. **(06 Marks)**

c. Find the angle between the planes, $x - y + z = 6$ and $2x + 3y + z = -5$. **(08 Marks)**

3 a. Find the unit normal to sum of the vectors $4i - j + 3k$ and $-2i + j - 2k$. **(06 Marks)**

b. Find $\vec{b} \times \left(\vec{a} \times \vec{c}\right)$, where $\vec{a} = i + j - k$, $\vec{b} = 2i - j + 2k$, $\vec{c} = 3i - j - k$. **(06 Marks)**

c. Find the angle between the vectors $\vec{a} = 5i - j + k$ and $\vec{b} = 2i - 3j + 6k$. **(08 Marks)**

4 a. A particle moves along the curve $\vec{r} = \cos 2ti + \sin 2tj + tk$. Find its velocity and acceleration. **(06 Marks)**

b. If $R = xi + yj + zk$, show that $\nabla \cdot R = 3$. **(06 Marks)**

c. Find the $F.\text{curl}\vec{F}$ where $\vec{F} = (x + y + 1)i + j - (x + y)k$. **(08 Marks)**

5 a. A particle moves along the curve $\vec{r} = (1 - t^3)i + (1 + t^2)j + (2t - 5)k$. Determine its velocity and acceleration. **(06 Marks)**

b. Show that the vector field, $\vec{F} = (3x + 3y + 4z)i + (x - 2y + 3z)j + (3x + 2y - z)k$ is solenoidal. **(06 Marks)**

c. Find $\text{curl}\vec{A}$ where $\vec{A} = xyi + y^2zj + z^2yk$. **(08 Marks)**

6 a. Find the Laplace transform of $1 + 3t^2 + 4e^{-3t}$. **(06 Marks)**

b. Find the Laplace transform of $\sin 4t + t^3$. **(06 Marks)**

c. Find the Laplace transform of $1 + e^{2t} + t^2 + \cosh t$. **(08 Marks)**

7 a. Find the Laplace transform of $e^{-2t} \sin 4t$. (06 Marks)

b. Find the Laplace transform of $t \sin 2t$. (06 Marks)

c. Find the inverse laplace transform of $\dfrac{s+2}{(s-1)(s-3)}$. (08 Marks)

8 a. Find the inverse Laplace transform of $\dfrac{1}{s^2 - 5s + 6}$ (06 Marks)

b. Find $L^{-1}\left\{\dfrac{1}{(s^2+1)(s^2+9)}\right\}$, by using convolution theorem. (06 Marks)

c. Solve $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 3y = e^{+t}$, $y(0) = y'(0) = 0$ by using Laplace transform method.

(08 Marks)

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