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15MATDIP41

Fourth Semester B.E. Degree Examination, Jan./Feb. 2023
Additional Mathematics – II

Time: 3 hrs.   Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1   a.   Find the rank of the matrix
$$A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$
by Elementary row transformation.   **(05 Marks)**

   b.   Solve the following system of equations by Gauss elimination method,
$x + y + z = 9$
$x - 2y + 3z = 8$
$2x + y - z = 3$ **(05 Marks)**

   c.   Find all the eigen values and eigen vectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.   **(06 Marks)**

OR

2   a.   Reduce the matrix
$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$
into echelon form and hence find its rank.   **(05 Marks)**

   b.   Find the inverse of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ using Cayley-Hamilton theorem.   **(05 Marks)**

   c.   Solve the following system of equations by Gauss elimination method.
$x + y + z = 9$
$2x + y - z = 0$
$2x + 5y + 7z = 52$ **(06 Marks)**

Module-2

3   a.   Solve $(D^3 - 6D^2 + 11D - 6) y = 0$ **(05 Marks)**

   b.   Solve $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 9y = 5e^{-2x}$ **(05 Marks)**

   c.   Solve $(D^2 + 1)y = \tan x$ by the method of variation of parameters.   **(06 Marks)**

OR

4   a.   Solve $(D^2 + 3D + 2)y = 2\cos 2x$ **(05 Marks)**

   b.   Solve $(D^2 + 2D + 1)y = 2x + x^2$ **(05 Marks)**

   c.   Solve $(D^2 - 2D + 5)y = e^{2x}$ by the method of undetermined coefficients.   **(06 Marks)**

Module-3

5   a. Find the Laplace transform of $\cos^2 3t$.   (05 Marks)
    b. Find $L[e^{-3t} \sin 2t]$ (05 Marks)
    c. Express $f(t) = \begin{cases} t^2 & 0 < t \le 2 \\ 4t & t > 2 \end{cases}$

        in terms of unit step function and hence find $L[f(t)]$.   (06 Marks)

OR

6   a. Find Laplace transform of $t^2 \cos at$ (05 Marks)
    b. Find $L\left\{ \dfrac{1-e^t}{t} \right\}$ (05 Marks)
    c. Find the Laplace transform of the function

        $f(t) = E \sin\left(\dfrac{\pi t}{w} \right)$, $0 < t < w$ given that $f(t + w) = f(t)$.   (06 Marks)

Module-4

7   a. Find the inverse Laplace transform of

        $\dfrac{s^2 - 3s + 4}{s^3}$ (05 Marks)
    b. Find the inverse Laplace transform of $\dfrac{s + 2}{s^2 - 4s + 13}$ (05 Marks)
    c. Solve the initial value problem

        $y'' + 4y' + 3y = e^{-t}$ with $y(0) = 1 = y'(0)$ using Laplace transform.   (06 Marks)

OR

8   a. Find $L^{-1}\left[\dfrac{(s+2)^3}{s^6} \right]$ (05 Marks)
    b. Find $L^{-1}\left[\dfrac{3s + 2}{(s - 2)(s + 1)} \right]$ (05 Marks)
    c. Solve by using Laplace transforms

        $\dfrac{d^2 y}{dt^2} + K^2 y = 0$ given that $y(0) = 2$, $y'(0) = 0$.   (06 Marks)

Module-5

9   a. If A and B are any two events, then prove that
        $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (05 Marks)
    b. A box contains 3 white, 5 black and 6 red balls. If a ball is drawn at random what is the
        probability that it is either red or white.   (05 Marks)
    c. Three machines A, B, C produces 50%, 30% and 20% of the items in a factory. The
        percentage of defective outputs are 3, 4, 5. If an item is selected at random, what is the
        probability that it is defective? What is the probability that it is from A?   (06 Marks)

OR

10 a. State and prove Baye's theorem.   (08 Marks)
    b. If A, B are two events having $P(A) = \dfrac{1}{2}$, $P(B) = \dfrac{1}{3}$ and $P(A \cap B) = \dfrac{1}{4}$. Compute the
        following :
        (i) $P(A/B)$ (ii) $P(B/A)$ (iii) $P(\overline{A}/\overline{B})$ (iv) $P(\overline{B}/\overline{A})$ (08 Marks)