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MATDIP401

Fourth Semester B.E. Degree Examination, Jan./Feb. 2023
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. A line makes angles $\alpha, \beta, \gamma, \delta$ with fur diagonals of a cube. Show that :
i) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$
ii) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = 8/3$. **(07 Marks)**

b. If (ℓ, m, n) and (ℓ_2, m_2, n_2) are the direction cosines of two lines subtending an angle θ between them, then prove that $\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$. **(07 Marks)**

c. Find the equation of the plane through (1, –2, 2), (–3, 1, –2) and perpendicular to the plane $2x - y - z + 6 = 0$. **(06 Marks)**

2 a. Find the image of the point (1, 1, 2) in the plane $2x + y + z - 3 = 0$. **(07 Marks)**

b. Find the symmetrical form of the line,
$2x + y - 3z = 3$
$3x + 2y - 5z - 5 = 0$. **(07 Marks)**

c. Show that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are coplanar. Find the coordinates of the point of intersection. **(06 Marks)**

3 a. Find the unit normal to both the vectors $4i - j + 3k$ and $-2i + j - 2k$. Find also the Sine of the angle between them. **(06 Marks)**

b. Prove that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$. **(08 Marks)**

c. i) Prove that $[\vec{b}\times\vec{c}, \vec{c}\times\vec{a}, \vec{a}\times\vec{b}] = [\vec{a}\,\vec{b}\,\vec{c}]^2$.

ii) Prove that $\vec{a}\times(\vec{b}\times\vec{c}) + \vec{b}\times(\vec{c}\times\vec{a}) + \vec{c}\times(\vec{a}\times\vec{b}) = 0$. **(06 Marks)**

4 a. A particle moves along the curve $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$. Determine the velocity and acceleration and their magnitudes at any time t. **(07 Marks)**

b. If $f = a\cos\omega t + b\sin\omega t$, where ω is a constant scalar, and a and b are constant vectors, show that $\left[f, \frac{df}{dt}, \frac{d^2f}{dt^2}\right] = 0$. **(07 Marks)**

c. Find the unit tangent vector of the space curve $\vec{r} = (1+t^3)i + 2t^3j + (2-t^3)k$. **(06 Marks)**

5 a. Find the angle between the surfaces,
$\phi_1 = x^2 + y^2 + z^2 - 9$
$\phi_2 = x^2 + y^2 - 3 - z$ at (2, –1, 2). **(07 Marks)**

b. Prove that Curl (grad F) = 0. **(06 Marks)**

c. Find constants a, b, c such that
$(x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is irrotational. **(07 Marks)**

6 a. If $f(t)=\begin{cases} t & 0<t<4 \\ 5 & t>4 \end{cases}$ find Lf(t). **(10 Marks)**

b. Find :

i) $L(\sin^2 3t)$

ii) $L(te^{-2t}\sin 4t)$

iii) $L\left(\frac{1-e^{-at}}{t}\right)$. **(10 Marks)**

7 a. If Lf(t) = F(s) then show that,

$L\left(\frac{f(t)}{t}\right)=\int_s^{\infty} F(s)ds$. **(10 Marks)**

b. Find :

i) $L^{-1}\left(\frac{s+5}{s^2-6s+13}\right)$

ii) $L^{-1}\left(\frac{3s+1}{(s-1)(s^2+1)}\right)$

iii) $L^{-1}\log\sqrt{s^2+1/s^2+4}$. **(10 Marks)**

8 a. Using the Laplace transform technique, solve

$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=3te^{-1}$

Given that $x=4$, $\frac{dx}{dt}=2$ when $t=0$. **(10 Marks)**

b. Solve the simultaneous equation using Laplace transforms :

$\frac{dx}{dy}+y=\sin t$

$\frac{dy}{dt}+x=\cos t$

Give that x = 1, t = 0 when t = 0. **(10 Marks)**

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