CBCS SCHEME

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17MAT31

Third Semester B.E. Degree Examination, Jan./Feb. 2023
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the Fourier series of $f(x) = x - x^2$ in $(-\pi, \pi)$. Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$ **(08 Marks)**

b. Find the half-range cosine series of $f(x) = \sin x$ in the interval $(0, \pi)$. **(06 Marks)**

c. Obtain the Fourier series up to first harmonic for the following data: **(06 Marks)**

x	0	1	2	3	4	5	6
f(x)	9	18	24	28	26	20	9

OR

2 a. Find the constant term and the first two harmonics in the Fourier series for f(x) given by the following table:

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(08 Marks)

b. Obtain the Fourier series of the function $f(x) = |x|$ in $(-\ell, \ell)$. **(06 Marks)**

c. Find the half-range sine series for the function $f(x) = \ell x - x^2$ in $(0, \ell)$. **(06 Marks)**

Module-2

3 a. Find the Fourier transform of the function

$$f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}, \text{ hence deduce that } \int_0^\infty \frac{(\sin x - x\cos x)}{x^3}\,dx = \frac{\pi}{4}.$$ **(08 Marks)**

b. Find the Fourier sine transform of $f(x) = e^{-|x|}$. Hence show that $\int_0^\infty \frac{(x \sin mx)}{1+x^2}\,dx = \frac{\pi e^{-m}}{2}$, $m > 0$. **(06 Marks)**

c. Find the inverse Z-transform of $\frac{3z^2 + z}{(5z-1)(5z+2)}$. **(06 Marks)**

OR

4 a. Find the Fourier Cosine transform of

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$$ **(06 Marks)**

b. Find the Z – transform of (i) $\cosh n\theta$ (ii) n^2 **(06 Marks)**

c. Solve $u_{n+2} + 3u_{n+1} + 2u_n = 3^n$, given $u_0 = 0$, $u_1 = 1$ by using Z-transforms. **(08 Marks)**

Module-3

5 a. Using Regula-Falsi method, find a real root (correct to three decimal places) of the equation $\cos x = 3x - 1$ that lies between 0.5 and 1. (Here x is in radians) **(08 Marks)**

b. Fit a straight line of the form $y = ax + b$ to the following data:

x	0	5	10	15	20	25
y	12	15	17	22	24	30

(06 Marks)

c. Calculate coefficient of correlation for the data below:

x	105	104	102	101	100	99	98	96	93	92
y	101	103	100	98	95	96	104	92	97	94

(06 Marks)

OR

6 a. If θ is the acute angle between the lines of regression, then show that $\tan\theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)$. Indicate the significance of the cases when $r = 0$ and $r = \pm 1$. **(08 Marks)**

b. Using Newton-Raphson method, find the real root of the equation $x\sin x + \cos x = 0$ near $x = \pi$, upto four decimal places. (Here x is in radians). **(06 Marks)**

c. Fit a parabola of the form $y = ax^2 + bx + c$ to the following data:

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

(06 Marks)

Module-4

7 a. The population of a town is given by the table

Year	1951	1961	1971	1981	1991
Population in thousands	19.96	39.65	58.81	77.21	94.61

Using Newton's forward and backward interpolation formula, calculate the increase in the population from the year 1955 to 1985. **(08 Marks)**

b. Using Newton's divided difference formula, represent f(x) as a polynomial in x for the following data:

x	−4	−1	0	2	5
f(x)	1245	33	5	9	1335

Hence find f(−2). **(06 Marks)**

c. By using Simpson's 1/3 rule, evaluate $\int_0^{0.6} e^{-x^2} dx$ by taking seven ordinates. **(06 Marks)**

OR

8 a. Using Newton's backward interpolation formula, find the interpolating polynomial for the function given by the following table

x	10	11	12	13
f(x)	21	23	27	33

Hence find f(12.5). **(08 Marks)**

b. Apply Lagrange's formula inversely to obtain a root of the equation f(x) = 0 given that f(30) = −30; f(34) = −13, f(38) = 3 and f(42) = 18. **(06 Marks)**

c. Evaluate $\int_4^{5.2} \log x\,dx$ by using Weddle's rule by taking seven ordinates. **(06 Marks)**

Module-5

9 a. Verify Green's theorem for $\int_C (xy + y^2)dx + x^2 dy$ where c is the closed curve made up of the line $y = x$ and the parabola $y = x^2$. **(08 Marks)**

b. If $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and s is the rectangular parallelepiped bounded by $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 1$, $z = 3$, evaluate $\iint_s \vec{F}.\hat{n}ds$. **(06 Marks)**

c. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(06 Marks)**

OR

10 a. Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half surface $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy - plane. **(08 Marks)**

b. Prove that geodesies of a plane are straight lines. **(06 Marks)**

c. Find the extremal of the functional $I = \int_0^{\pi/2} \left[(y')^2 - y^2 + 4y\cos x\right]dx$, given that $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$. **(06 Marks)**

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